

STRUCTURAL ANALYSIS OF TOWER B3 ACCORDING TO SNIP II-7-81*

FOR

Project: “Technical Expertise and develop Detailed Technical Design for
conservation/restoration works of Bender Fortress”

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1. INTRODUCTION

1.1. Objectives of structural analysis

The task consists of performing the structural analysis of the Bender Fortress - Tower B3. Construction element will be calculated individually. The process consists of calculation of characteristic and design loads and performing the static and dynamic analysis, including the determination of the dynamic proprieties of tower based on the Moldavian design standards.

1.2. Documentary basis of structural analysis

As reference documents for structural analysis were used the following:

- [1] **“Studio Berlucchi” srl** – Technical expertise and develop detailed technical design for conservation and restoration works of Bender Fortress (Phase I)
- [2] **Nicoara I.; Bogdevici O.** Report on geological data Tighina Fortress
- [3] NCM E.02.02:2016. Fiabilitatea în construcții.
- [4] NCM F.03.02-2005. Proiectarea construcțiilor cu pereți din zidărie.
- [5] СНиП 2.01.07-85. Нагрузки и воздействия.
- [6] СНиП II-7-81*. Строительство в сейсмических районах.
- [7] СНиП 2.02.01-83. Основания зданий и сооружений.

Technical-scientific literature used:

- **Atanasiu M. Gabriela** “Structural Dynamics”, *Vasilie Goldis University Press*, Arad 2000
- **Гордеев В.Н. и др.** “Нагрузки и воздействия на здания и сооружения”, *Издательство Ассоциации Строителей Вузов* – 2000
- **Birbrae r A.N.** “*Seismic Analysis of Structures.*” - St. Petersburg: Nauka, 1998. -255 p.
- СВОД ПРАВИЛ. *Трубы Промышленные Дымовые. Правила проектирования*, министерство строительстваи жилищно-коммунального хозяйства российской федерации - Москва 2016

1.3. Category of importance

Normative “NCM E.02.02:2016. Fiabilitatea în construcții.” (Reliability in Construction) does not mention a clear category of importance for historical monuments or architectural heritage. But given the historical significance of the studied objective; structure could be classified as CC-3 level of importance (Hight level), group 2 (p.2.5, 2.12) with minimum value of reliability coefficient $\gamma_n = 1.1$.

2. ANALITICAL PART

2.1. Description of the analyzed object

The B3 tower will be modeled and analyzed as cantilever (see SNiP II-7-81*).

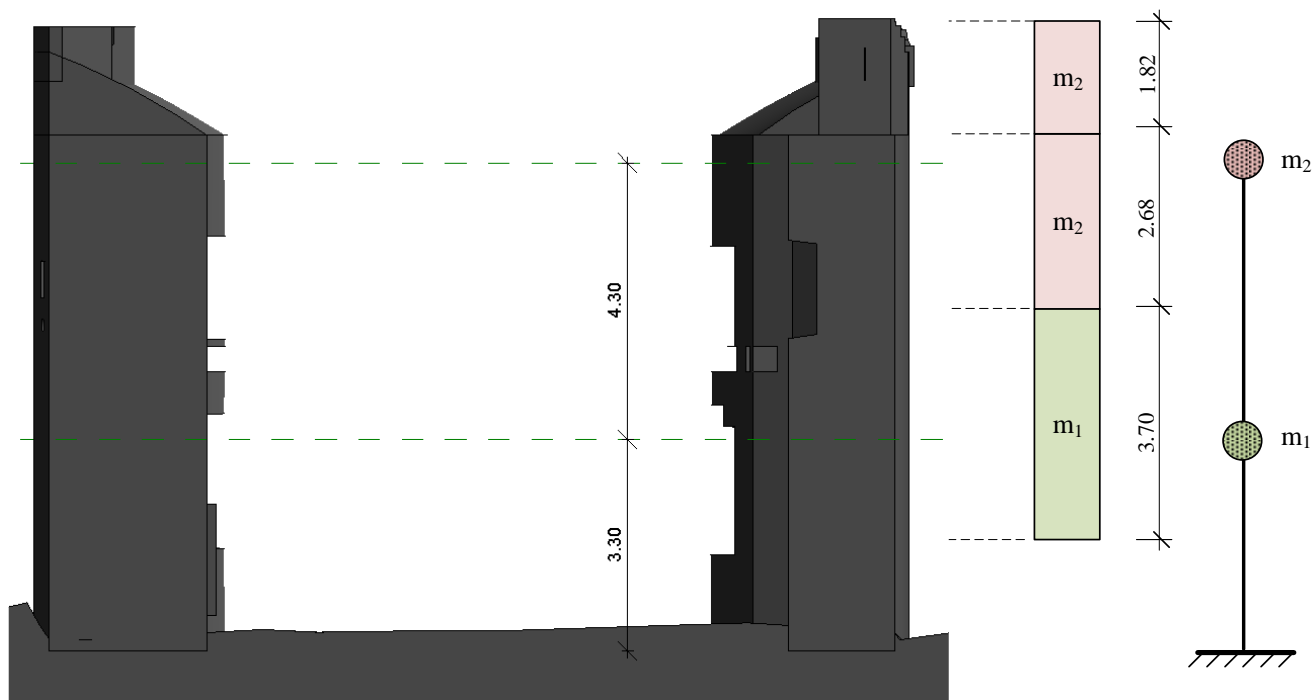


Figure 1 Analyzed model of the B3 tower

Unlike A6 tower, for B3 tower is decided to select a 2 DOF model for few reasons like the building high and the consolidation model proposed for this type of structure.

Information about the construction region

- Air temperature:
 - minimum air temperature – (-) 41.4 °C;
 - maximum air temperature – (+) 31.2 °C;
- Area of the characteristic value of the snow load on the ground – I.
The characteristic value of the snow load on the ground per 1 m^2 – $s_0 = 0,5 kPa$.
- Area of the characteristic value of the wind pressure on the ground – II.
The characteristic value of the wind pressure – $w_0 = 0,3 kPa$.
- Site seismicity – 7 grades according to MSK-64 scale.

2.2. Structural characteristic of building

2.2.1. Rigidity

For the analysis of tower B3 a section that have the following geometrical form was taken (see Annex 1):

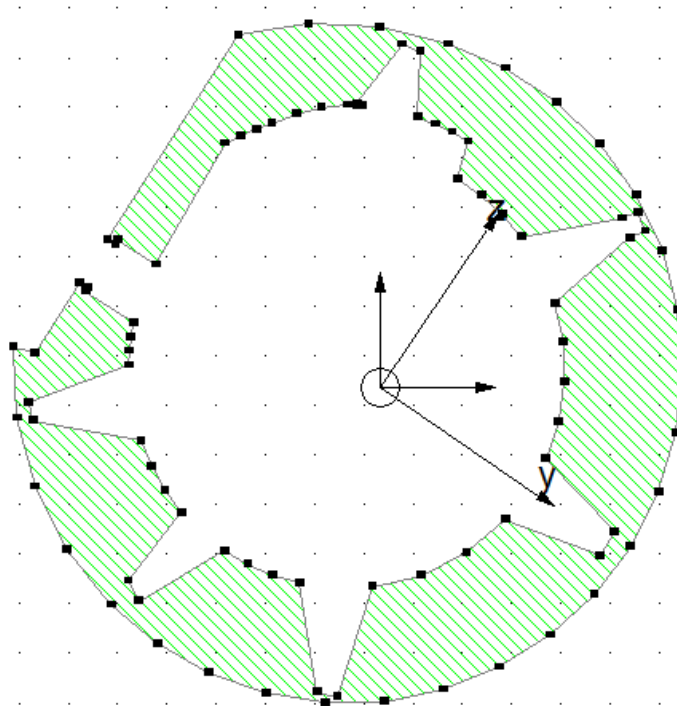


Figure 2 The cross section for tower B3

The elastic modulus of masonry was computed by using expression (6) given in [4]:

$$E_0 = \alpha R_u$$

where α – the elastic characteristic of unreinforced masonry and R_u is assigned value of $2R$; R – design strength of masonry to compression taken from table 18 and 19 of NCM F.03.02-2005. Proiectarea construcțiilor cu pereți din zidărie.

$$E_0 = 350 \cdot 2 \cdot 1.3 = 910 \text{ (MPa)}$$

Deformational modulus is calculated by using following expression provided in [4]:

$$E = K_n E_0 = 0.8 \cdot 910 = 728 \text{ (MPa)}$$

2.2.2. Loads on structure

Table 1 Loads on structure

Description	Unit	Normative value	Safety coefficient γ_f	Design Value	Note
Permanent load					
Masonry wall ($\delta = 2450 \text{ mm}$, $A_w = 60.302 \text{ m}^2$, $\rho = 1900 \text{ kg/m}^3$)	kN/m	1123.58	1.3	1460.6	NCM F.03.02-2005

The design value first mass is:

$$Q_1 = Q_{1,perm} \cdot 0.9 + Q_{1,qvc} \cdot 0.8 + Q_{1,var} \cdot 0.5 = 4864.0 \text{ kN}$$

$$Q_{1,perm} = h_1 \cdot q_{mw} = 5404.44 \text{ kN}$$

$$Q_{1,qvc} = 0 \text{ kN}$$

$$Q_{1,var} = 0 \text{ kN}$$

$$Q_2 = Q_{2,perm} \cdot 0.9 + Q_{2,qvc} \cdot 0.8 + Q_{2,var} \cdot 0.5 = 4056.72 \text{ kN}$$

$$Q_{2,perm} = h_2 \cdot q_{mw} + q_{wall} = 4507.461 \text{ kN}$$

$$Q_{2,qvc} = 0 \text{ kN}$$

$$Q_{2,var} = 0 \text{ kN}$$

where: h_i – design height for computing mass m_1, m_2, m_3

q_{mw} – design value of masonry wall in kN/m

$q_{wall} = 592.89 \text{ kN}$ – is the weight of the upper part of tower (see Figure 1)

Masses for first design model are:

$$m_1 = \frac{Q_1}{g} = 4.96 \cdot 10^5 (kg); \quad m_2 = \frac{Q_2}{g} = 5.511 \cdot 10^5 (kg);$$

where $g = 9.807 \text{ m/s}^2$

2.3. Calculus

The equation system of motion is obtained after writing the equilibrium of all forces acting at a time t on the masses $m_i, i = 1..N$.

$$\{F_i(t)\} + \{F_a(t)\} + \{F_e(t)\} = \{F(t)\} \quad (1)$$

where

$\{F_i(t)\} = [M]\{\ddot{u}(t)\}$ – is the inertia forces vector,

$\{F_a(t)\} = [C]\{\dot{u}(t)\}$ – is the damping forces vector,

$\{F_e(t)\} = [K]\{u(t)\}$ – is the elastic linear forces vector,

$\{F(t)\}$ – is the vector of the applied forces on structure

The system (1) can be written as:

$$[M]\{\ddot{u}_i(t)\} + [C]\{\dot{u}_i(t)\} + [K]\{u_i(t)\} = \{F_i(t)\} \quad (2)$$

where

$\{\ddot{u}_i(t)\}$ – is the acceleration column vector,

$\{\dot{u}_i(t)\}$ – is the velocities column vector,

$\{u_i(t)\}$ – is the displacements column vector,

$[M]$ – is the mass matrix

$[C]$ – is the damping matrix

$[K]$ – is the stiffness matrix.

2.3.1. Development of flexibility and stiffness matrix

By applying the unit force for every DOF of system, the flexibility matrix is created:

$$U = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} = \begin{bmatrix} 1.391 & 4.11 \\ 4.11 & 16.99 \end{bmatrix} \cdot 10^{-11} \left(\frac{m}{N} \right)$$

The stiffness matrix $[K]$, can be calculated as follows:

$$[K] = U^{-1} = \begin{bmatrix} 25.2 & -6.097 \\ -6.097 & 2.063 \end{bmatrix} \cdot 10^{10} \left(\frac{N}{m} \right)$$

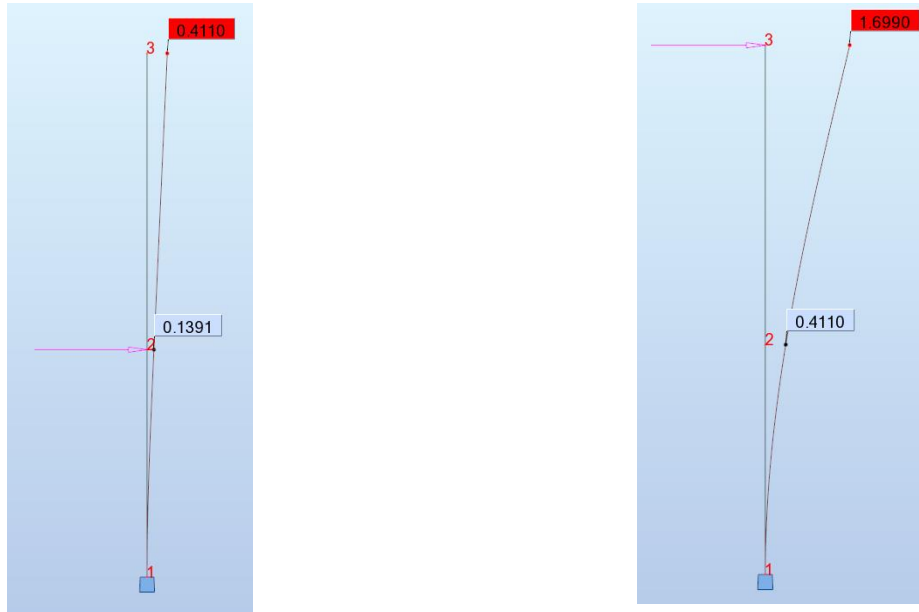


Figure 3 Unit force applied to each of DOF

2.3.2. Computation of modal characteristics

The natural frequencies and corresponding mode shapes can be determined after solving the equation:

$$(K - \omega^2 M)\Phi = 0 \quad (3)$$

Equation (3) is generalized eigenvalue problem. The quantities ω^2 are the eigenvalues i.e. the squares of frequencies; the corresponding displacement vectors Φ represent the corresponding mode of vibration of the dynamic model (known as the eigenvectors or modal shapes). The eigenvalue problem is solved using the following relationship:

$$M\Phi = M\Phi\Omega^2 \quad (3)$$

Solving equation (3), we obtain spectral matrix and mode shape matrix:

$$\Omega = \begin{bmatrix} 5.355 \cdot 10^5 & 0 \\ 0 & 1.014 \cdot 10^4 \end{bmatrix} \left(\left(\frac{rad}{s} \right)^2 \right)$$

$$\Phi = \begin{bmatrix} 1 & 1 \\ 4.051 & -0.222 \end{bmatrix}$$

Knowing spectral matrix, we can compute frequencies and periods of structure:

$$\omega = \begin{bmatrix} 100.698 & 0 \\ 0 & 731.779 \end{bmatrix} \left(\frac{rad}{s} \right)$$

$$T = \begin{bmatrix} 0.062 & 0 \\ 0 & 0.009 \end{bmatrix} (s)$$

3. RESULTS

3.1. Dynamic proprieties of structure

Table 2 Dynamic proprieties of strucutre

	Parameter	Tower B3
Mode I	Frequency	$\omega = 100.698 (s^{-1})$
	Period	$T = 0.062 (s)$
	Modal shape vector	$\Phi_1 = \begin{Bmatrix} 1 \\ 4051 \end{Bmatrix}$
	MPMR*	74.526 %
Mode II	Frequency	$\omega = 731.779 (s^{-1})$
	Period	$T = 0.009 (s)$
	Modal shape vector	$\Phi_2 = \begin{Bmatrix} 1 \\ -0.222 \end{Bmatrix}$
	MPMR*	25.473 %

NOTE: MPMR* - Modal participating mass ratio.

Modal participating mass ratio (MPMR) – represents the part of the total mass which responds to earthquake motion in each mode.

3.2. Seismic force according to SNiP II-7-81*

3.2.1. Determination of constant values

Seismic force according to SNiP II-7-81*, applied in k and that correspondes to vibrations mode i is determinate as follows:

$$S_{ik} = K_1 \cdot K_2 \cdot Q_k \cdot A \cdot \beta_i \cdot K_\psi \cdot \eta_{ik} \quad (4)$$

where:

$K_1 = 0.25$ – from table 3 from SNiP II-7-81*

$K_2 = 1$ – from table 4 from SNiP II-7-81*

$K_\psi = 1$ – from table 6 from SNiP II-7-81*

$A = 0.1$ – for intensity of site 7 grade MSK-64, see p.2.5 from SNiP II-7-81*

η_{ik} – form coefficient calculated by equation (6)

Knowing constant values, the equation (4) can be written:

$$S_{ik} = K \cdot Q_k \cdot \beta_i \cdot \eta_{ik} \quad (5)$$

where:

$$K = K_1 \cdot K_2 \cdot K_\psi \cdot A = 0.25 \cdot 1 \cdot 1 \cdot 0.1 = 0.05$$

3.2.2. Determination of dynamic coefficients

In accordance with p.2.6 of SNiP II-7-81* for soil category III and vibration periods $T_i < 0.1$, the dynamic coefficient is computed by following expression:

$$\beta_i = 17 \cdot T_i + 1$$

but value of β_i in all cases should not be less than 0.8.

The dynamic coefficients of structure are:

$$\beta_1 = 17 \cdot 0.062 + 1 = 2.054$$

$$\beta_2 = 17 \cdot 0.009 + 1 = 1.153$$

3.2.3. Form coefficients

Relation for computing form coefficients can be found in SNiP II-7-81*, p.2.7 as follows:

$$\eta_{ik} = \frac{X_i(x_k) \sum_{j=1}^n Q_j X_i(x_j)}{\sum_{j=1}^n Q_j X_i^2(x_j)} \quad (6)$$

where:

$X_i(x_k)$ – displacements of a building with its own vibrations in the i mode at the considered point k and all points j , where in accordance with the calculation scheme its weight is assumed concentrated;

Q_j – weight of building or structure, referred to point j determined taking into account the design load on the structure

- For 1 mode of vibration

$$\eta_{11} = \frac{X_1(x_1)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 0.286$$

$$\eta_{12} = \frac{X_1(x_2)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 1.159$$

- For 2 mode of vibration

$$\eta_{21} = \frac{X_2(x_1)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 0.714$$

$$\eta_{22} = \frac{X_2(x_2)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = -0.159$$

Verifying condition of correct determination of coefficients:

$$\sum_{k=1}^j \eta_{ik} \cong 1$$

- For first weight

$$\eta_{11} + \eta_{21} = 1.00$$

- For second weight

$$\eta_{12} + \eta_{22} = 1.00$$

3.2.4. Determination of seismic force

- For I mode of vibration

$$S_{11} = K \cdot Q_1 \cdot \beta_1 \cdot \eta_{11} = 142.857 \text{ (kN)}$$

$$S_{12} = K \cdot Q_2 \cdot \beta_1 \cdot \eta_{12} = 643.074 \text{ (kN)}$$

- For II mode of vibration

$$S_{21} = K \cdot Q_1 \cdot \beta_2 \cdot \eta_{21} = 200.218 \text{ (kN)}$$

$$S_{22} = K \cdot Q_2 \cdot \beta_2 \cdot \eta_{22} = -49.42 \text{ (kN)}$$

The sum vector for at each DOF, see equation (8) from СНиП II-7-81*:

$$S_1 = \sqrt{S_{11}^2 + S_{21}^2} = 245.958 \text{ (kN)}$$

$$S_2 = \sqrt{S_{12}^2 + S_{22}^2} = 644.97 \text{ (kN)}$$

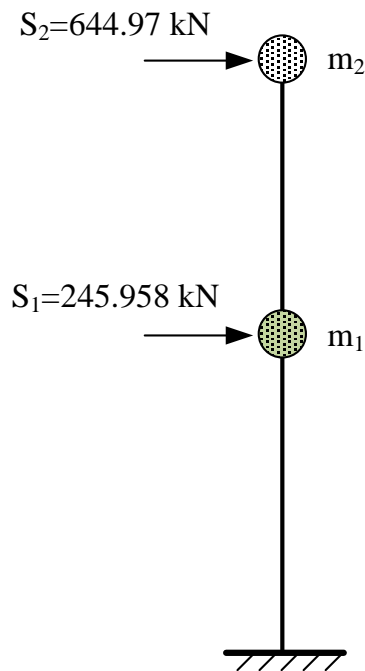


Figure 4 The seismic force for B3 Tower

4. CONCLUSION

The structural analysis of B3 tower according to SNiP-7-81* was made. As conclusion the following can be stated:

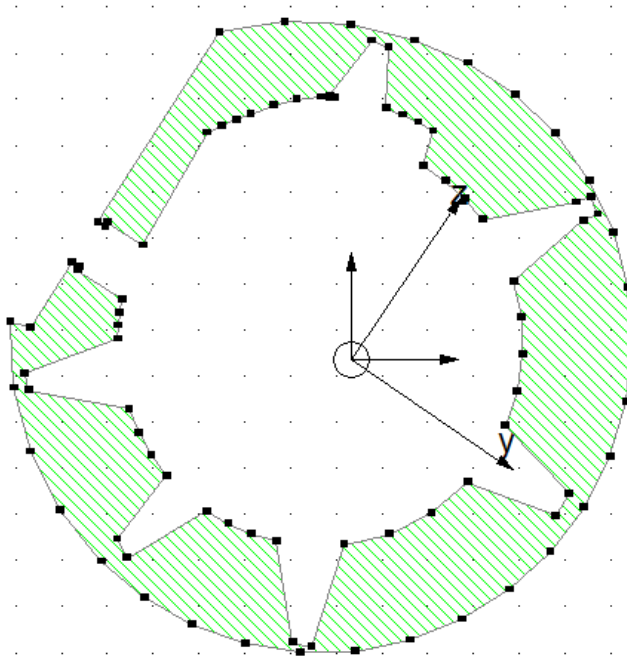
1. For tower B3 was used 2 DOF model with lumped masses applied in points where the consolidation works will be made. This model will provide the highest possible seismic force at studied points.
2. It should be noted that only self-weight of structure was taken into account, without possible installation of floors, roofs and/or other loads that could appear in case of installation of other structural elements.
3. One can certainly affirm that tower B3 that is part of Bender Fortress represents an architectural landmark. This being said it should be noted that use of “behavior coefficient” $k_1 = 0.25$ is not justifiable. However, SNiP II-7-81* does not offer an alternative to use coefficient $k_1 \geq 0.25$.
4. From “**Studio Berlucchi**” srl – *Technical expertise and develop detailed technical design for conservation and restoration works of Bender Fortress (Phase I)* it is pointed that tower B3 is not loaded by the decks or roofs, this makes it vulnerable because of box-like effect. Thus, to escape this effect the tower should be “squeezed”.
5. “**Studio Berlucchi**” srl proposal for intervention consists of an external hoop of the masonry with stainless steel strands inserted inside mortar joints. This intervention will help to restrain lateral deformations of the tower, i.e. will increase its compressive strength due to volumetric stress.
6. In case of the proposed intervention, the minimum area necessary for external hoop is:

$$A_{1n} \geq \frac{S_1}{\gamma_c R_y} = \frac{245.958}{240 \cdot 1} = 10.248 \text{ (cm}^2\text{)}$$
$$A_{2n} \geq \frac{S_2}{\gamma_c R_y} = \frac{644.97}{240 \cdot 1} = 26.874 \text{ (cm}^2\text{)}$$

where $R_y = 240 \text{ MPa}$ – is yield strength for steel class C245 according to GOST 27772-88. Taking into account the historical significance, the safety coefficient $\gamma_s \geq 1.2$ should be considered.

7. Apart from this should be considered installing structural monitoring systems that will help to analyse the structural “health” and to monitor the building behavior, changing of dynamic proprieties during an earthquake and other parameters.

ANNEX 1 Section proprieties



General results

Area	A = 60.302 m ²
Center of gravity	Y _c = 1094328.66 mm Z _c = -1105647.91 mm
Perimeter	S = 42700.97 mm
Base material	STEEL E = 199948023.75 kPa den = 7849.05 kg/m ³ WU = 473310.62 kgf/m

Principal system

Angle	alpha = -34.5 Deg
Moments of inertia	I _x = 59.392 m ⁴ I _y = 1104.141 m ⁴ I _z = 893.813 m ⁴
Radii of inertia	i _y = 4279.05 mm i _z = 3849.98 mm
Shear areas	A _y = 36.900 m ² A _z = 35.592 m ²

Central system

Moments of inertia	I _{yc} = 1036.564 m ⁴
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Radii of inertia

$I_{zc} = 961.390 \text{ m}^4$
 $I_{ycz} = 98.218 \text{ m}^4$

Maximum distances

$i_{yc} = 4146.04 \text{ mm}$
 $i_{zc} = 3992.87 \text{ mm}$

Arbitrary system

$V_{yc} = 6143.39 \text{ mm}$
 $V_{pyc} = 7445.07 \text{ mm}$
 $V_{zc} = 7327.21 \text{ mm}$
 $V_{pzc} = 6351.58 \text{ mm}$

System position

$y_{c'} = 1094328.66 \text{ mm}$
 $z_{c'} = -1105647.91 \text{ mm}$

Angle = 0.0 Deg

Moments of inertia

$I_{y'} = 1036.564 \text{ m}^4$
 $I_{z'} = 961.390 \text{ m}^4$
 $I_{y'z'} = 98.218 \text{ m}^4$

Radii of inertia

$i_{yc} = 4146.04 \text{ mm}$
 $i_{zc} = 3992.87 \text{ mm}$

First moments of area

$S_{y'} = -0.000 \text{ m}^3$
 $S_{z'} = 0.000 \text{ m}^3$

Maximum distances

$V_{y'} = 6143.39 \text{ mm}$
 $V_{py'} = 7445.07 \text{ mm}$
 $V_{z'} = 7327.21 \text{ mm}$
 $V_{pz'} = 6351.58 \text{ mm}$