STRUCTURAL ANALYSIS OF TOWER A6 ACCORDING TO SNIP II-7-81*

FOR

Project: “Technical Expertise and develop Detailed Technical Design for conservation/restoration works of Bender Fortress”

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1. INTRODUCTION

1.1. Objectives of structural analysis

The task consists of performing the structural analysis of the Bender Fortress - Tower A6. Construction element will be calculated individually. The process consists of calculation of characteristic and design loads and performing the static and dynamic analysis, including the determination of the dynamic proprieties of tower based on the Moldavian design standards.

1.2. Documentary basis of structural analysis

As reference documents for structural analysis were used the following:

[1] **“Studio Berlucchi” srl** – Technical expertise and develop detailed technical design for conservation and restoration works of Bender Fortress (Phase I)

[2] **Nicoara I.; Bogdevici O.** Report on geological data Tighina Fortress

[3] NCM E.02.02:2016. Fiabilitatea în construcții.

[4] NCM F.03.02-2005. Proiectarea construcțiilor cu pereți din zidărie.

[5] СНиП 2.01.07-85. Нагрузки и воздействия.

[6] СНиП II-7-81*. Строительство в сейсмических районах.

[7] СНиП 2.02.01-83. Основания зданий и сооружений.

Technical-scientific literature used:

- **Atanasiu M. Gabriela** “Structural Dynamics”, *Vasilie Goldis University Press*, Arad 2000
- **Гордеев В.Н**. и др. “Нагрузки и воздействия на здания и сооружения”, *Издательство Ассоциации Строителиных Вузов* – 2000
- **Birbrae r A.N.** “*Seismic Analysis of Structures.*” - St. Petersburg: Nauka, 1998. -255 p.
- СВОД ПРАВИЛ. *Трубы Промышленные Дымовые. Правила проектирования*, министерство строительстваи жилищно-коммунального хозяйствароссийской федерации - Москва 2016

1.3. Category of importance

Normative “NCM E.02.02:2016. Fiabilitatea în construcții.” (Reliability in Construction) does not mention a clear category of importance for historical monuments or architectural heritage. But given the historical significance of the studied objective; structure could be classified as CC-3 level of importance (Hight level), group 2 (p.2.5, 2.12) with minimum value of reliability coefficient $\gamma_n = 1.1$.

2. ANALITICAL PART

2.1. Description of the analyzed object

The A6 tower will be modeled and analyzed as cantilever (see SNiP II-7-81*). Two models with 3 and 2 degree of freedom will be compared.

The first model with 3 degree of freedom will be derived from number of floors of tower. The lumped massed of the A6 tower will be concentrated at elevations +22.37, +25.56, +30.36.

The second model with 2 degrees of freedom will concentrate masses in $\frac{1}{3} \div \frac{2}{3}$ from the upper part of structure where consolidation works should be made.

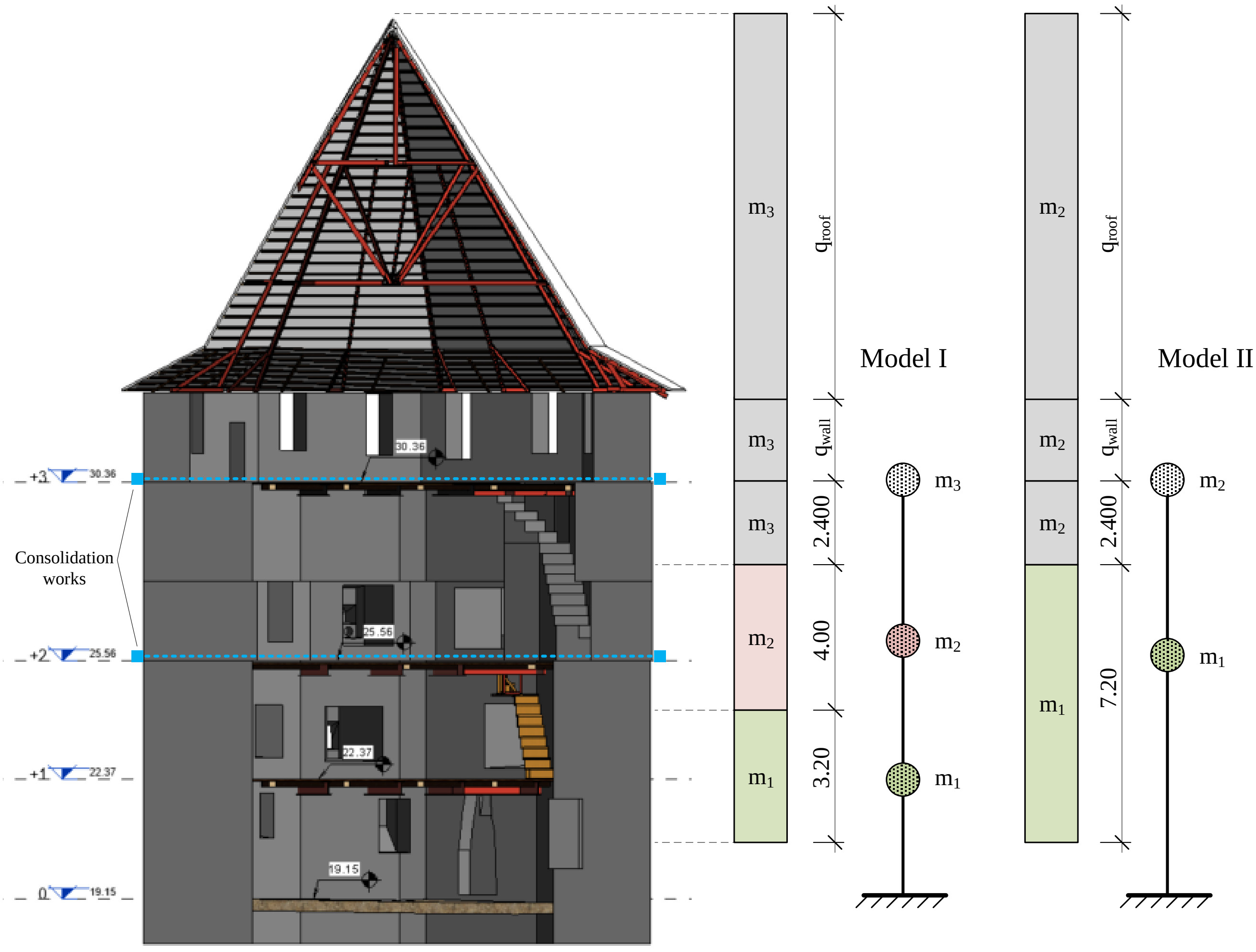


Figure 1 Analyzed model of the A6 tower

The results from the both analyzed models should be compared. After having all the outputs, the most appropriate model for should be chosen in for consolidation works

2.2. Information about the construction region

- Air temperature:
  - minimum air temperature – (-) 41.4 °C;
  - maximum air temperature – (+) 31.2 °C;
- Area of the characteristic value of the snow load on the ground – I.
  The characteristic value of the snow load on the ground per 1 m^2– $s_0 = 0{,}5\ kPa$.
- Area of the characteristic value of the wind pressure on the ground – II.
  The characteristic value of the wind pressure – $w_0 = 0{,}3\ kPa$.
- Site seismicity – 7 grades according to MSK-64 scale.

2.3. Structural characteristic of building

2.3.1. Rigidity

For the analysis of tower A6 a section that have the following geometrical form was taken (see Annex 1):

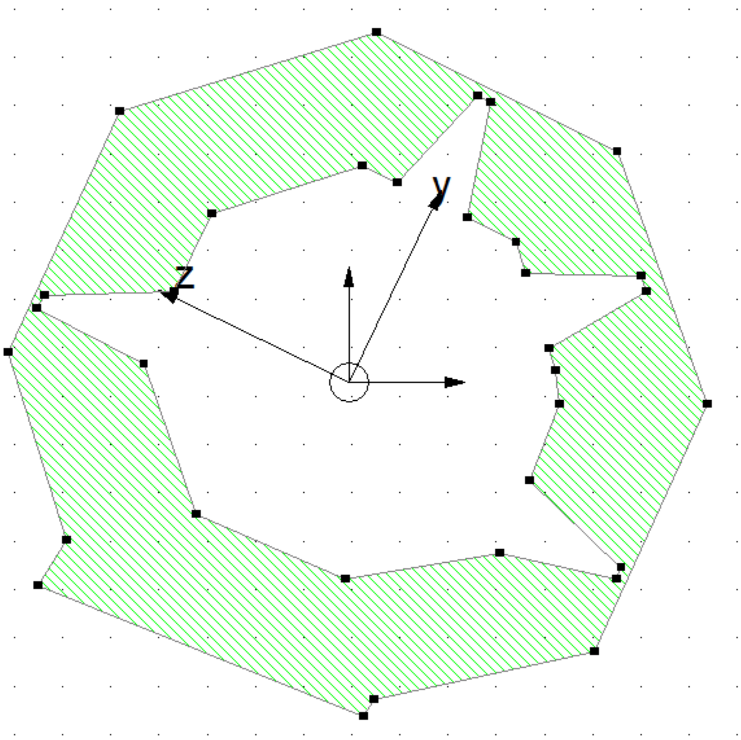


Figure 2 The cross section for tower A6

The elastic modulus of masonry was computed by using expression (6) given in [4]:

$$E_0 = \alpha R_u$$

where α – the elastic characteristic of unreinforced masonry and R_u is assigned value of $2R$; R – design strength of masonry to compression taken from table 18 and 19 of NCM F.03.02-2005. Proiectarea construcțiilor cu pereți din zidărie.

$$E_0 = 350 \cdot 2 \cdot 1.3 = 910\ (MPa)$$

Deformational modulus is calculated by using following expression provided in [4]:

$$E = K_n E_0 = 0.8 \cdot 910 = 728\ (MPa)$$

2.3.2. Loads on structure

Table 1 Loads on structure

Description	Unit	Normative value	Safety coefficient γ_f	Design Value	Note
Permanent load					
Wood deck ($\delta = 35\ mm$, $\rho = 850\ kg/m^3$)	kN/m^2	0.292	1.3	0.379	СНиП 2.01.07-85, tab. 1
Wood beam ($bxh = 50x150\ mm, \rho = 850\ kg/m^3$)	kN/m	0.063	1.3	0.0819	СНиП 2.01.07-85, tab. 2
Steel beam (profile IPE 300)	kN/m	0.56	1.05	0.59	СНиП 2.01.07-85, tab. 1
Masonry wall ($\delta = 2550\ mm$, $A_w = 84.643\ m^2$, $\rho = 1900\ kg/m^3$)	kN/m	1577.1	1.3	2050.3	NCM F.03.02-2005
Equivalent roof load (See annex)	kN/m^2	0.789	1.2	0.947	
Live load (P_t)					
Quasi-permanent (p_{qvc})					
Quasi-permanent on slab	kN/m^2	1.4	1.3	1.82	СНиП 2.01.07-85, tab. 1 and 3
Variable Load (p_{var})					
Variable load on slab		2.6	1.2	3.12	СНиП 2.01.07-85, tab. 1 and 3

NOTE: Snow load will not be considered in calculation, see Annex 2 from [5].

The design value first floor is:

$$Q_1 = Q_{1,perm} \cdot 0.9 + Q_{1,qvc} \cdot 0.8 + Q_{1,var} \cdot 0.5 = 6128.514\ kN$$
$$Q_{1,perm} = p_{wd} \cdot A_{wd} + l_{wb1} \cdot q_{wb} + l_{sb1} \cdot q_{sb} + h_1 \cdot q_{mw} = 6624.16\ kN$$
$$Q_{1,qvc} = p_{qvc} \cdot A_{wd} = 100.65\ kN$$
$$Q_{1,var} = p_{var} \cdot A_{wd} = 172.5\ kN$$

$$Q_2 = Q_{2,perm} \cdot 0.9 + Q_{2,qvc} \cdot 0.8 + Q_{2,var} \cdot 0.5 = 8380\ kN$$
$$Q_{2,perm} = p_{wd} \cdot A_{wd} + l_{wb2} \cdot q_{wb} + l_{sb2} \cdot q_{sb} + h_2 \cdot q_{mw} = 7604.51\ kN$$
$$Q_{2,qvc} = p_{qvc} \cdot A_{wd} = 99.19\ kN$$
$$Q_{2,var} = p_{var} \cdot A_{wd} = 172.5\ kN$$

$$Q_3 = Q_{3,perm} \cdot 0.9 + Q_{3,qvc} \cdot 0.8 + Q_{3,var} \cdot 0.5 = 6011.7\ kN$$
$$Q_{3,perm} = p_{wd} \cdot A_{wd} + l_{wb3} \cdot q_{wb} + l_{sb3} \cdot q_{sb} + p_r \cdot A + h_3 \cdot q_{mw} + q_{wall} = 6494.37\ kN$$
$$Q_{3,qvc} = p_{qvc} \cdot A_{wd} = 100.65\ kN$$
$$Q_{3,var} = p_{var} \cdot A_{wd} = 172.5\ kN$$

where: p_{wd} – design load of wood deck in kN/m^2

A_{wd} – area of wood deck 55.3 m^2

l_{wb} – length of wood beams in m for 1st,2ed and 3rd floor

q_{wb} – design load of wood beam in kN/m

l_{sb} – length of steel beams in m for 1st,2ed and 3rd floor3

q_{sb} – design load of steel beam in kN/m

h_i – design height for computing mass m_1,m_2,m_3

q_{ms} – design value of masonry wall in kN/m

$q_{wall} = 1430.05\ kN$ – is the weight of the upper part of tower (see Figure 1)

Masses for first design model are:

$$m_1 = \frac{Q_1}{g} = 6.249 \cdot 10^5 (kg);\ m_2 = \frac{Q_2}{g} = 7.754 \cdot 10^5 (kg);\ \ m_3 = \frac{Q_3}{g} = 6.622 \cdot 10^5 (kg)$$

Masses for second design model are:

$$m_1 = \frac{Q_1}{g} + \frac{Q_2}{g} = 14.00 \cdot 10^5 (kg)\ \ m_2 = \frac{Q_3}{g} = 6.622 \cdot 10^5 (kg)$$

where $g = 9.807\ m/s^2$

2.4. Calculus

The equation system of motion is obtained after writing the equilibrium of all forces acting at a time t on the masses m_i, $i = 1..N$.

$$\{F_i(t)\} + \{F_a(t)\} + \{F_e(t)\} = \{F(t)\} \qquad (1)$$

where

$\{F_i(t)\} = [M]\{\ddot{u}(t)\}$ – is the inertia forces vector,

$\{F_a(t)\} = [C]\{\dot{u}(t)\}$ – is the damping forces vector,

$\{F_e(t)\} = [K]\{u(t)\}$ – is the elastic linear forces vector,

$\{F(t)\}$ – is the vector of the applied forces on structure

The system (1) can be written as:

$$[M]\{\ddot{u}_i(t)\} + [C]\{\dot{u}_i(t)\} + [K]\{u_i(t)\} = \{F_i(t)\} \qquad (2)$$

where

$\{\ddot{u}_i(t)\}$ – is the acceleration column vector,

$\{\dot{u}_i(t)\}$ – is the velocities column vector,

$\{u_i(t)\}$ -is the displacements column vector,

$[M]$ – is the mass matrix

$[C]$ – is the damping matrix

$[K]$ – is the stiffness matrix.

2.4.1. Development of flexibility and stiffness matrix

2.4.1.1. First Design Model

By applying the unit force for every DOF of system, the flexibility matrix is created:

$$U = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix} = \begin{bmatrix} 1.03 & 2.56 & 4.86 \\ 2.56 & 8.13 & 17.25 \\ 4.86 & 17.25 & 43.46 \end{bmatrix} \cdot 10^{-11} \left(\frac{m}{N}\right)$$

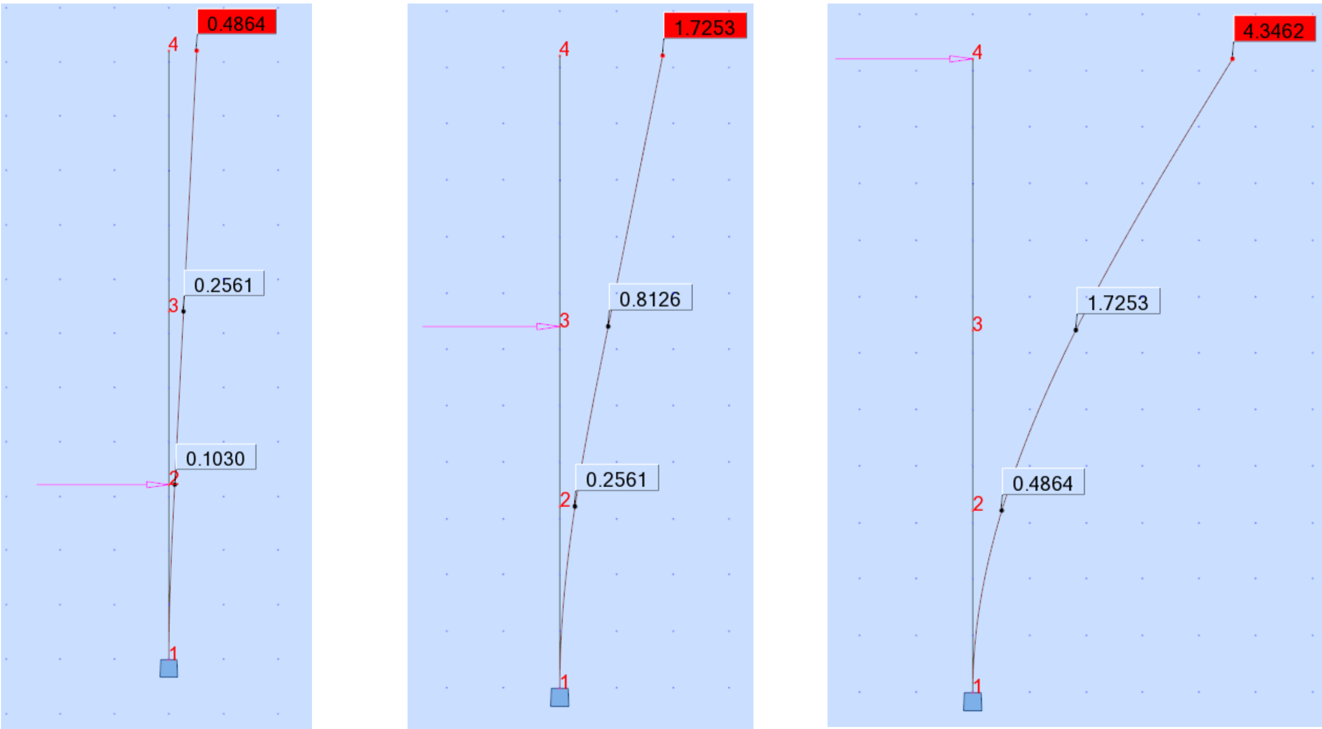


Figure 3 Unit force applied to each of DOF for I model case

The stiffness matrix $[K]$, can be calculated as follows:

$$[K] = U^{-1} = \begin{bmatrix} 5.6739 & -2.79 & 0.4729 \\ -2.79 & 2.1513 & -0.5419 \\ 0.4729 & -0.5419 & 0.1852 \end{bmatrix} \cdot 10^{11} \left(\frac{N}{m}\right)$$

2.4.1.2. Second Design Model

By applying the unit force for every DOF of system, the flexibility matrix is created:

$$U = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} = \begin{bmatrix} 8.13 & 17.25 \\ 17.25 & 43.46 \end{bmatrix} \cdot 10^{-11} \left(\frac{m}{N}\right)$$

The stiffness matrix $[K]$, can be calculated as follows:

$$[K] = U^{-1} = \begin{bmatrix} 7.832 & -3.109 \\ -3.109 & 1.464 \end{bmatrix} \cdot 10^{10} \left(\frac{N}{m}\right)$$

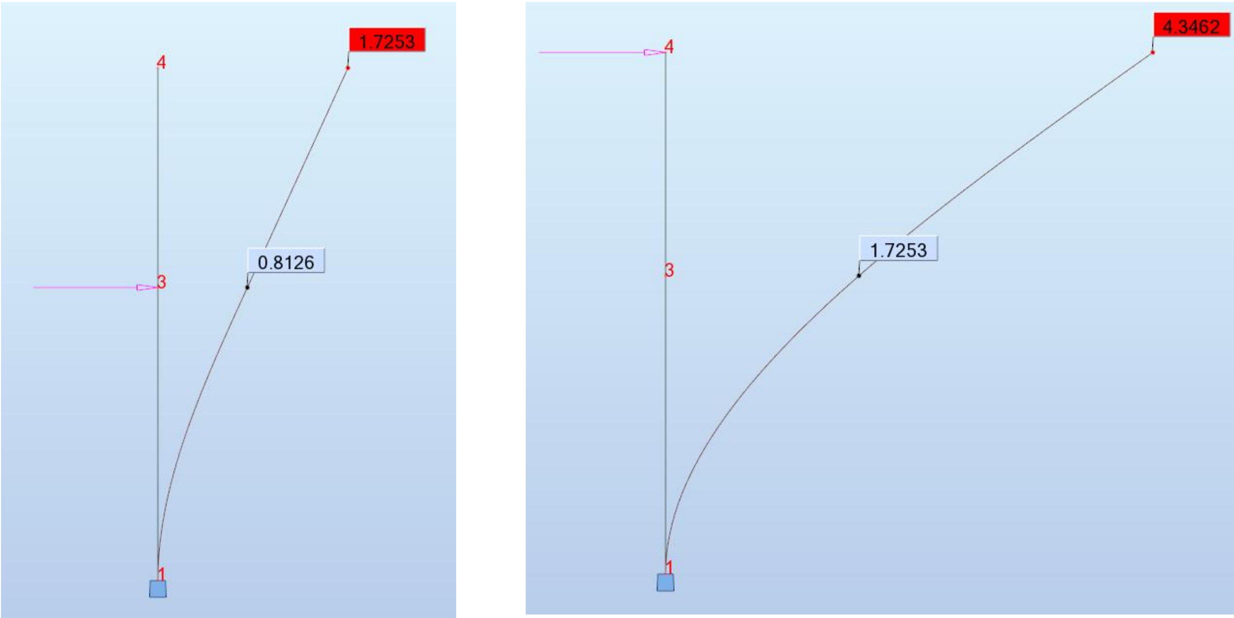


Figure 4 Unit force applied to each of DOF for II model case

2.4.2. Computation of modal characteristics

The natural frequencies and corresponding mode shapes can be determined after solving the equation:

$$(K - \omega^2 M)\Phi = 0 \qquad (3)$$

Equation (3) is generalized eigenvalue problem. The quantities ω^2 are the eigenvalues i.e. the squares of frequencies; the corresponding displacement vectors Φ represent the corresponding mode of vibration pf the dynamic model (known as the eigenvectors or modal shapes). The eigenvalue problem is solved using the following relationship:

$$M\Phi = M\Phi\Omega^2 \qquad (3)$$

2.4.2.1. First Design Model

Solving equation (3), we obtain spectral matrix and mode shape matrix:

$$\Omega = \begin{bmatrix} 1.112 \cdot 10^6 & 0 & 0 \\ 0 & 9.875 \cdot 10^4 & 0 \\ 0 & 0 & 2.888 \cdot 10^3 \end{bmatrix} \left(\left(\frac{rad}{s} \right)^2 \right)$$

$$\Phi = \begin{bmatrix} 1 & 1 & 1 \\ 3.456 & 1.658 & -0.44 \\ 8.427 & -0.908 & 0.099 \end{bmatrix}$$

Knowing spectral matrix, we can compute frequencies and periods of structure:

$$\omega = \begin{bmatrix} 53.74 & 0 & 0 \\ 0 & 314.245 & 0 \\ 0 & 0 & 1054.51 \end{bmatrix} \left(\frac{rad}{s} \right)$$

$$T = \begin{bmatrix} 0.1169 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.006 \end{bmatrix} (s)$$

2.4.2.2. Second Design Model

Solving equation (3), we obtain spectral matrix and mode shape matrix:

$$\Omega = \begin{bmatrix} 7.547 \cdot 10^4 & 0 \\ 0 & 2.575 \cdot 10^3 \end{bmatrix} \left(\left(\frac{rad}{s} \right)^2 \right)$$

$$\Phi = \begin{bmatrix} 1 & 1 \\ 2.403 & -0.88 \end{bmatrix}$$

Knowing spectral matrix, we can compute frequencies and periods of structure:

$$\omega = \begin{bmatrix} 50.744 & 0 \\ 0 & 274.718 \end{bmatrix} \left(\frac{rad}{s} \right)$$

$$T = \begin{bmatrix} 0.124 & 0 \\ 0 & 0.023 \end{bmatrix} (s)$$

3. RESULTS

3.1. Dynamic proprieties of structure

Table 2 Dynamic proprieties of strucutre

	Parameter	I Model Case	II Model Case
Mode I	Frequency	$\omega = 53.74\ (s^{-1})$	$\omega = 50.744\ (s^{-1})$
	Period	$T = 0.117\ (s)$	$T = 0.124\ (s)$
	Modal shape vector	$\Phi_1 = \begin{Bmatrix} 1 \\ 3.456 \\ 8.427 \end{Bmatrix}$	$\Phi_1 = \begin{Bmatrix} 1 \\ 2.403 \end{Bmatrix}$
	MPMR*	67.256 %	83.07 %
Mode II	Frequency	$\omega = 314.245\ (s^{-1})$	$\omega = 274.718\ (s^{-1})$
	Period	$T = 0.02\ (s)$	$T = 0.023\ (s)$
	Modal shape vector	$\Phi_2 = \begin{Bmatrix} 1 \\ 1.658 \\ -0.908 \end{Bmatrix}$	$\Phi_2 = \begin{Bmatrix} 1 \\ -0.88 \end{Bmatrix}$
	MPMR*	25.166 %	16.94 %
Mode III	Frequency	$\omega = 1054.51\ (s^{-1})$	-
	Period	$T = 0.006\ (s)$	-
	Modal shape vector	$\Phi_3 = \begin{Bmatrix} 1 \\ -0.44 \\ 0.099 \end{Bmatrix}$	-
	MPMR*	7.589 %	-

NOTE: MPMR*- Modal participating mass ratio.

Modal participating mass ratio (MPMR) – represents the part of the total mass which responds to earthquake motion in each mode.

3.2. Seismic force according to SNiP II-7-81*

3.2.1. Determination of constant values

Seismic force according to SNiP II-7-81*, applied in k and that correspondes to vibrations mode i is determinate as follows:

$$S_{ik} = K_1 \cdot K_2 \cdot Q_k \cdot A \cdot \beta_i \cdot K_\Psi \cdot \eta_{ik} \quad (4)$$

where:

$K_1 = 0.25$ – from table 3 from SNiP II-7-81*

$K_2 = 1$ – from table 4 from SNiP II-7-81*

$K_\Psi = 1$ – from table 6 from SNiP II-7-81*

$A = 0.1$ – for intensity of site 7 grade MSK-64, see p.2.5 from SNiP II-7-81*

η_{ik} – form coefficient calculated by equation (6)

Knowing constant values, the equation (4) can be written:

$$S_{ik} = K \cdot Q_k \cdot \beta_i \cdot \eta_{ik} \quad (5)$$

where:

$$K = K_1 \cdot K_2 \cdot K_\Psi \cdot A = 0.25 \cdot 1 \cdot 1 \cdot 0.1 = 0.05$$

3.2.2. Determination of dynamic coefficients

In accordance with p.2.6 of SNiP II-7-81* for soil category III and vibration periods $T_i < 0.1$, the dynamic coefficient is computed by following expression:

$$\beta_i = 17 \cdot T_i + 1$$

if vibration period $0.1 < T_i < 0.5$ then dynamic coefficient is $\beta_i = 2.7$, for other cases if $T_i > 0.5$ the coefficient is computed using following relation:

$$\beta_i = \frac{1.35}{T_i}$$

but value of β_i in all cases should not be less than 0.8.

3.2.2.1. First Design Model

The dynamic coefficients of structure are:

$$\beta_1 = 2.7$$
$$\beta_2 = 17 \cdot 0.02 + 1 = 1.34$$
$$\beta_3 = 17 \cdot 0.006 + 1 = 1.102$$

3.2.2.2. Second Design Model

The dynamic coefficients of structure are:

$$\beta_1 = 2.7$$
$$\beta_2 = 17 \cdot 0.023 + 1 = 1.391$$

3.2.3. Form coefficients

Relation for computing form coefficients can be found in SNiP II-7-81*, p.2.7 as follows:

$$\eta_{ik} = \frac{X_i(x_k)\sum_{j=1}^{n} Q_j X_i(x_j)}{\sum_{j=1}^{n} Q_j X_i^2(x_j)} \tag{6}$$

where:

$X_i(x_k)$ – displacements of a building with its own vibrations in the i mode at the considered point k and all points j, where in accordance with the calculation scheme its weight is assumed concentrated;

Q_j – weight of building or structure, referred to point j determined taking into account the design load on the structure

3.2.3.1. First Design Model

- For 1 mode of vibration

$$\eta_{11} = \frac{X_1(x_1)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 0.156$$

$$\eta_{12} = \frac{X_1(x_2)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 0.54$$

$$\eta_{13} = \frac{X_1(x_3)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 1.316$$

- For 2 mode of vibration

$$\eta_{21} = \frac{X_2(x_1)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 0.396$$

$$\eta_{22} = \frac{X_2(x_2)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 0.657$$

$$\eta_{23} = \frac{X_2(x_3)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = -0.36$$

- For 3 mode of vibration

$$\eta_{31} = \frac{X_3(x_1)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 0.447$$

$$\eta_{32} = \frac{X_3(x_2)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = -0.197$$

$$\eta_{33} = \frac{X_3(x_3)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 0.044$$

Verifying condition of correct determination of coefficients:

$$\sum_{k=1}^{j} \eta_{ik} \cong 1$$

- For first weight

$$\eta_{11} + \eta_{21} + \eta_{31} = 0.998$$

- For second weight

$$\eta_{12} + \eta_{22} + \eta_{32} = 1.00001$$

- For third weight

$$\eta_{13} + \eta_{23} + \eta_{33} = 1.00005$$

3.2.3.2. Second Design Model

- For 1 mode of vibration

$$\eta_{11} = \frac{X_1(x_1)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 0.573$$
$$\eta_{12} = \frac{X_1(x_2)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 1.377$$

- For 2 mode of vibration

$$\eta_{21} = \frac{X_2(x_1)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = 0.427$$
$$\eta_{22} = \frac{X_2(x_2)[Q_1 \cdot X_1(x_1) + Q_2 \cdot X_1(x_2) + Q_3 \cdot X_1(x_3)]}{Q_1 \cdot X_1^2(x_1) + Q_2 \cdot X_1^2(x_2) + Q_3 \cdot X_1^2(x_3)} = -0.376$$

Verifying condition of correct determination of coefficients:

$$\sum_{k=1}^{j} \eta_{ik} \cong 1$$

- For first weight

$$\eta_{11} + \eta_{21} = 0.999$$

- For second weight

$$\eta_{12} + \eta_{22} = 1.00001$$

3.2.4. Determination of seismic force

3.2.4.1. First Design Model

- For I mode of vibration

$$S_{11} = K \cdot Q_1 \cdot \beta_1 \cdot \eta_{11} = 129.165\ (kN)$$
$$S_{12} = K \cdot Q_2 \cdot \beta_1 \cdot \eta_{12} = 553.904\ (kN)$$
$$S_{13} = K \cdot Q_3 \cdot \beta_1 \cdot \eta_{13} = 1153.452\ (kN)$$

- For II mode of vibration

$$S_{21} = K \cdot Q_1 \cdot \beta_2 \cdot \eta_{21} = 162.785\ (kN)$$

$$S_{22} = K \cdot Q_2 \cdot \beta_2 \cdot \eta_{22} = 334.901\ (kN)$$

$$S_{23} = K \cdot Q_3 \cdot \beta_2 \cdot \eta_{23} = -156.633\ (kN)$$

- For III mode of vibration

$$S_{31} = K \cdot Q_1 \cdot \beta_3 \cdot \eta_{31} = 150.921\ (kN)$$

$$S_{32} = K \cdot Q_2 \cdot \beta_3 \cdot \eta_{32} = -82.398\ (kN)$$

$$S_{33} = K \cdot Q_3 \cdot \beta_3 \cdot \eta_{33} = 15.833\ (kN)$$

- The sum vector for at each DOF, see equation (8) from СНиП II-7-81*:

$$S_1 = \sqrt{S_{11}^2 + S_{21}^2 + S_{31}^2} = 256.826\ (kN)$$

$$S_2 = \sqrt{S_{12}^2 + S_{22}^2 + S_{32}^2} = 652.501\ (kN)$$

$$S_3 = \sqrt{S_{13}^2 + S_{23}^2 + S_{33}^2} = 1164.146\ (kN)$$

3.2.4.2. Second Design Model

- For I mode of vibration

$$S_{11} = K \cdot Q_1 \cdot \beta_1 \cdot \eta_{11} = 1061.66\ (kN)$$

$$S_{12} = K \cdot Q_2 \cdot \beta_1 \cdot \eta_{12} = 1207.2\ (kN)$$

For II mode of vibration

$$S_{21} = K \cdot Q_1 \cdot \beta_2 \cdot \eta_{21} = 408.172\ (kN)$$

$$S_{22} = K \cdot Q_2 \cdot \beta_2 \cdot \eta_{22} = -169.725\ (kN)$$

The sum vector for at each DOF, see equation (8) from СНиП II-7-81*:

$$S_1 = \sqrt{S_{11}^2 + S_{21}^2 + S_{31}^2} = 1137.42\ (kN)$$

$$S_2 = \sqrt{S_{12}^2 + S_{22}^2 + S_{32}^2} = 1219.073\ (kN)$$

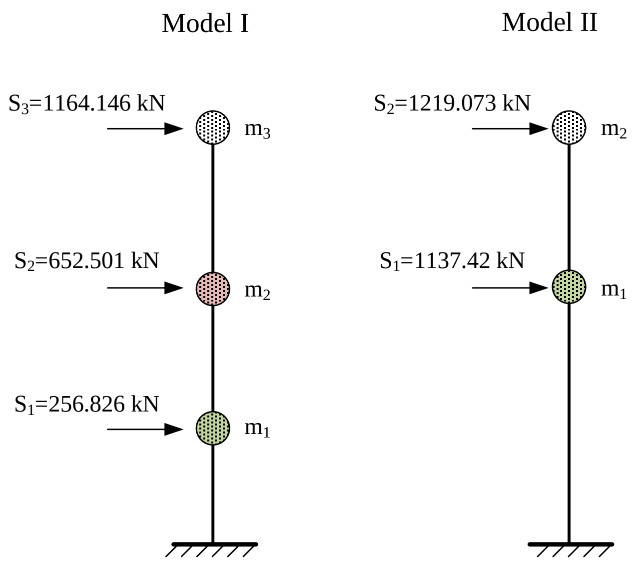


Figure 5 The seismic force for I and II design model

4. CONCLUSION

The structural analysis of two models of A6 tower according to SNiP II-7-81* was made. As conclusion the following be stated:

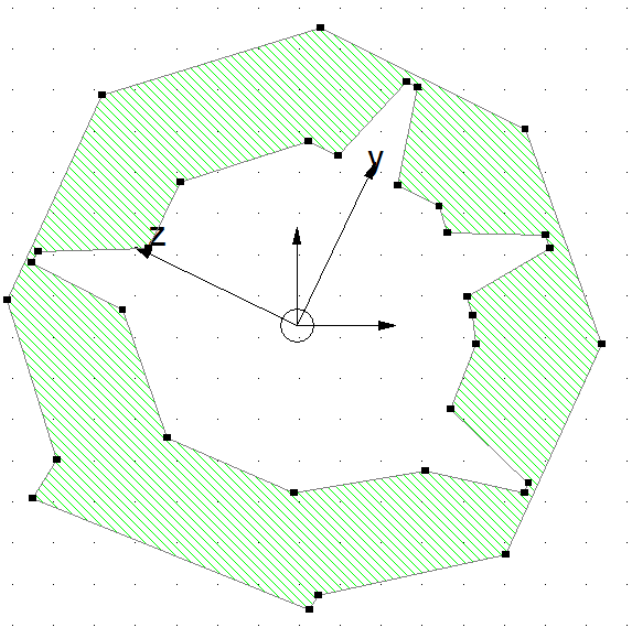
1. As can be noticed from both cases, in the first mode shape the participating mass ratio is higher than in other modal shapes. Thus, for I design model the modal participating mass ratio is 67.26 % and for II design model the MPMR is 83.07% from total mass. This indicates that the most reliable design model is with 2 degrees of freedom.
2. From results, one can observe that seismic force from I design model at point “1” is significantly lower than seismic force from other two points. This suggests us, that this point could be omitted in favor of 2 DOF model, i.e. II design model.
3. From reviewed scientific literature one can affirm that the most significant damage in towers is occurred at upper part of structure, $\frac{1}{3} \div \frac{2}{3}$ from top. Thus, this is another argument for II design model.
4. One can certainly affirm that tower A6 that is part of Bender Fortress represents an architectural landmark. This being said it should be noted that use of “behavior coefficient” $k_1 = 0.25$ is not justifiable. However, SNiP II-7-81* does not offer an alternative to use coefficient $k_1 \geq 0.25$.
5. From **“Studio Berlucchi” srl** – *Technical expertise and develop detailed technical design for conservation and restoration works of Bender Fortress (Phase I)* it is pointed out a longitudinal crack along North-East façade. This fact implies that due an seismic event could be triggered failure mechanism, so the consolidation works are required.
6. As intervention could be proposed to inject inside the cracks mortar. Along with mortar injection in cracks, should be considered installing tie rods at the most critical points of building that will ensure overall stiffness of structure due to an earthquake.
7. **“Studio Berlucchi” srl** proposal on installing tie rods at second and third level could be applied. In this case the minimum area necessary for one tie rod for the levels 2 and 3 are:

$$A_{1n} \geq \frac{S_1}{2\gamma_c R_y} = \frac{1137.42}{2 \cdot 240 \cdot 1} = 23.7\ (cm^2)$$
$$A_{2n} \geq \frac{S_2}{2\gamma_c R_y} = \frac{1219.073}{2 \cdot 240 \cdot 1} = 25.39\ (cm^2)$$

   where $R_y = 240\ MPa$ – is yield strength for steel class C245 according to GOST 27772-88. Taking into account the historical significance, the safety coefficient $\gamma_s \geq 1.2$ should be considered.

8. Apart from this should be considered installing structural monitoring systems that will help to analyse the structural “health” and to monitor the building behavior, changing of dynamic proprieties during an earthquake and other parameters.

ANNEX 1 Section proprieties



General results

Area

A = 84.643 m2

Center of gravity

Yc = 1029938.99 mm
Zc = 1065272.47 mm

Perimeter

S = 45327.53 mm

Principal system

Angle

alpha = 64.2 Deg

Moments of inertia

Ix = 154.176 m4
Iy = 1431.515 m4
Iz = 1374.868 m4

Radii of inertia

iy = 4112.46 mm
iz = 4030.27 mm

Shear areas

Ay = 35.692 m2
Az = 59.004 m2

Central system

Moments of inertia

Iyc = 1385.634 m4
Izc = 1420.749 m4
Iyczc = -22.225 m4

Radii of inertia

iyc = 4046.02 mm
izc = 4096.97 mm

Maximum distances

Vyc = 7442.15 mm

Vpyc	=	7067.75 mm
Vzc	=	7241.48 mm
Vpzc	=	6887.46 mm

Arbitrary system

System position

yc'	=	1029938.99 mm	Angle =	0.0 Deg
zc'	=	1065272.47 mm		

Moments of inertia

Iy'	=	1385.634 m4
Iz'	=	1420.749 m4
Iy'z'	=	-22.226 m4

Radii of inertia

iyc	=	4046.02 mm
izc	=	4096.97 mm

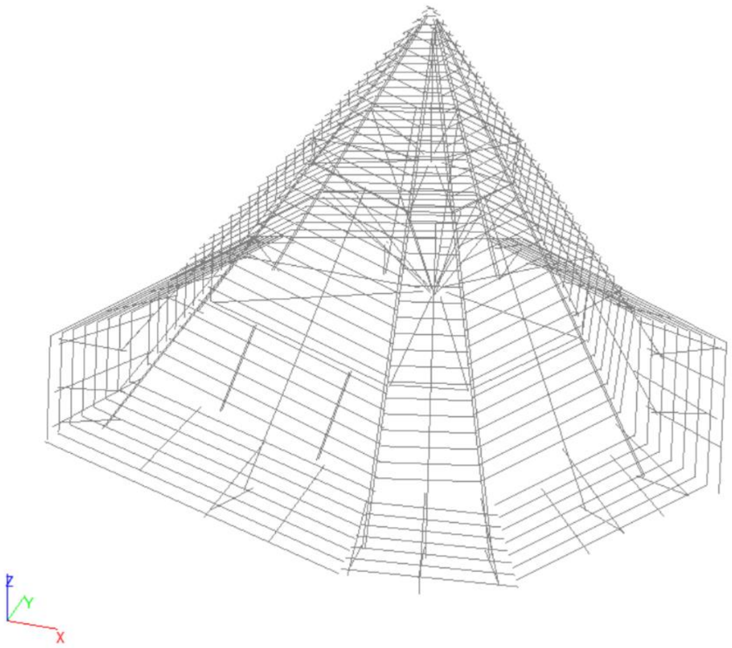
First moments of area

Sy'	=	0.000 m3
Sz'	=	0.000 m3

Maximum distances

Vy'	=	7442.15 mm
Vpy'	=	7067.75 mm
Vz'	=	7241.48 mm
Vpz'	=	6887.46 mm

ANNEX 2 Roof weight



Load case		Sum of external loads					
		X	Y	Z	U_X	U_Y	U_Z
		Tone	Tone	Tone	T*м	T*м	T*м
1	Self – weight	-4.941e-009	1.311e-008	6.81	0	0	0

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