EXPLANATION NOTE FOR NEW ROOF STRUCTURE

FOR

Project: “Technical Expertise and develop Detailed Technical Design for conservation/restoration works of Bender Fortress”

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Table of contents

1. INTRODUCTION

1.1. Objectives of structural analysis

The task consists of performing the structural analysis of new roof structure for the Bender Fortress towers. The structure of new roof is formed by main wooden beams with two stiffening rings around main beams. The process consists of calculation of characteristic and design loads and performing the analysis for verifying if the cross section of structural elements satisfied conditions prescribed in the Moldavian design standards.Documentary basis of structural analysis

As reference documents for structural analysis were used the following:

[1] **“Studio Berlucchi” srl** – Technical expertise and develop detailed technical design for conservation and restoration works of Bender Fortress (Phase I)

[2] **Nicoara I.; Bogdevici O.** Report on geological data Tighina Fortress

[3] NCM E.02.02:2016. Fiabilitatea în construcții.

[4] NCM F.05.01-2007. Proiectarea construcțiilor din lemn.

[5] СНиП 2.01.07-85. Нагрузки и воздействия.

[6] СНиП II-7-81*. Строительство в сейсмических районах.

[7] СНиП 2.02.01-83. Основания зданий и сооружений.

Technical-scientific literature used:

- **Atanasiu M. Gabriela** “Structural Dynamics”, *Vasilie Goldis University Press*, Arad 2000
- **Гордеев В.Н**. и др. “Нагрузки и воздействия на здания и сооружения”, *Издательство Ассоциации Строителиных Вузов* – 2000
- **Birbrae r A.N.** “*Seismic Analysis of Structures.*” - St. Petersburg: Nauka, 1998. -255 p.
- СВОД ПРАВИЛ. *Трубы Промышленные Дымовые. Правила проектирования*, министерство строительстваи жилищно-коммунального хозяйствароссийской федерации - Москва 2016

1.2. Category of importance

Normative “NCM E.02.02:2016. Fiabilitatea în construcții.” (Reliability in Construction) places the roof structure in class CC-2, level of importance is normal. The minimum value of reliability coefficient for importance is $\gamma_n = 1$.

2. ANALITICAL PART

2.1. Description of the analyzed object

New roof structure is made of timber beams with two stiffening rings around it and covered with wooden shingles.

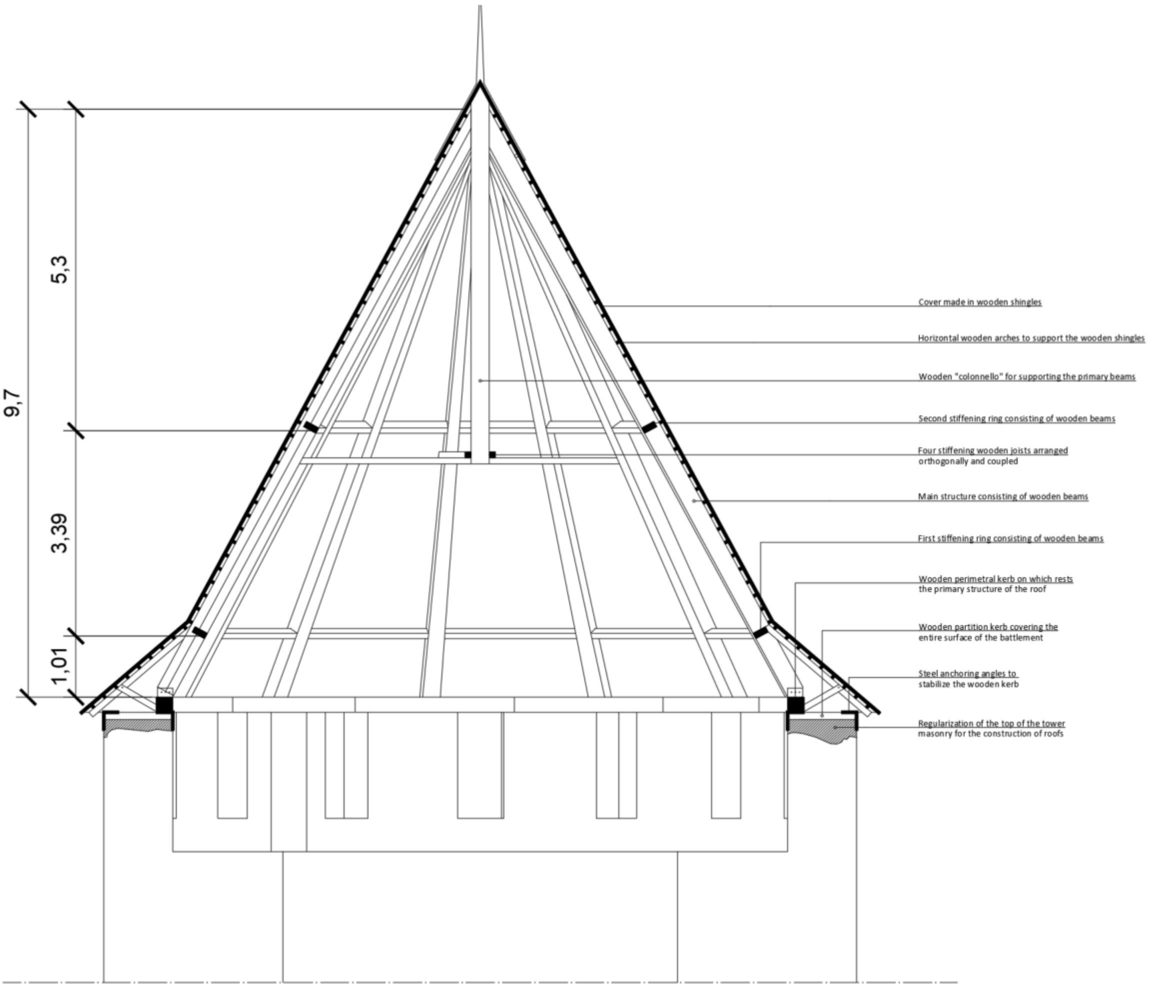

Figure 1 Roof section

Information about the construction region

- Air temperature:
  - minimum air temperature – (-) 41.4 °C;
  - maximum air temperature – (+) 31.2 °C;
- Area of the characteristic value of the snow load on the ground – I.
  The characteristic value of the snow load on the ground per 1 m^2– $s_0 = 0,5\ kPa$.
- Area of the characteristic value of the wind pressure on the ground – II.
  The characteristic value of the wind pressure – $w_0 = 0,3\ kPa$.
- Site seismicity – 7 grades according to MSK-64 scale.

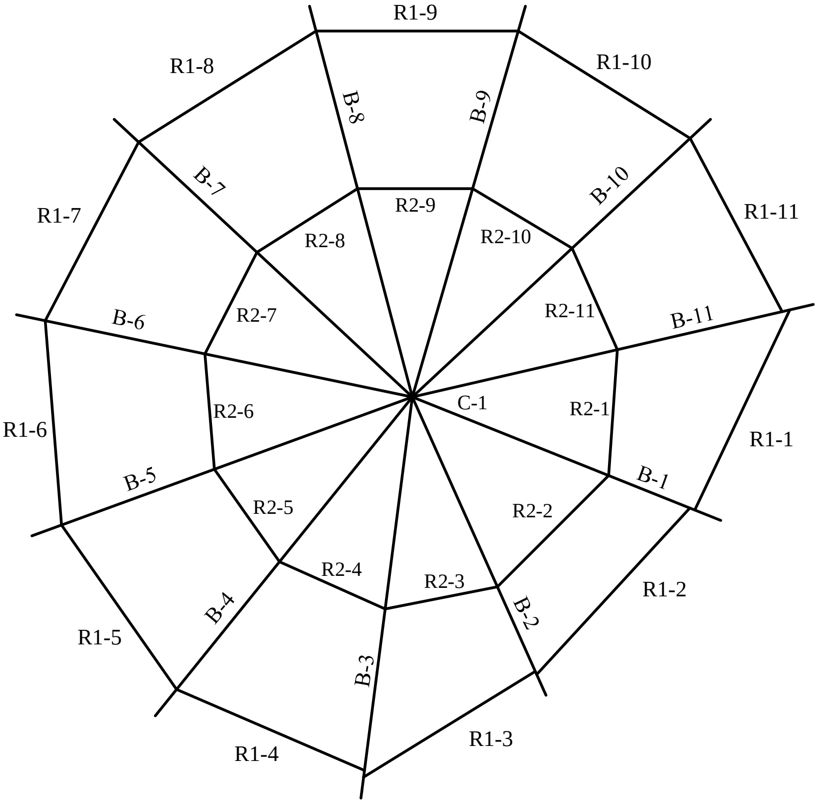

Figure 2 Plan view of roof structure

Beams $B-1$ and $B-2$, compared to other main beams are placed with a larger angle $\cong 68°$. This is caused due to irregular plan shape of the tower.

2.2. Structural characteristic of building

2.2.1. Rigidity

Element	Section form	Sections dimensions (mm)
$R1-1 \div R1-11$	Rectangular $(b\ x\ h)$	$100\ x\ 200$
$R2-1 \div R2-11$	Rectangular $(b\ x\ h)$	$100\ x\ 200$
$B-1 \div B-11$	Rectangular $(b\ x\ h)$	$300\ x\ 300$
$C-1$	Circular (d)	$\emptyset 300$

* see annex for detailed proprieties

The elastic modulus for design of timber elements is adopted according to [4]:

$$E = 300R_{C_{II}} = 300 \cdot 10.4 = 3120\ (MPa)$$

, where $R_{c_{II}} = R_{(Tab)c_{II}} \cdot m_{sp} = 13 \cdot 0.8 = 10.4\ (MPa)$ for rectangular section elements made of 2nd sort of timber.

m_{sp} – coefficient equal to 0.8 for fir tree.

2.2.2. Loads on structure

The weight of cover will be generalized in linear load on beams (l_b – beam length):

$$q_c = \frac{0.816 \frac{kN}{m^2} \cdot 21.68\ m^2}{l_b} = 1.67\ kN/m$$

Table 1 Load on roof structure

Description	Unit	Normative value	Safety coefficient γ_f	Design value	Note
PERMANENT LOAD					
Wooden shingles	kN/m^2	0.628	1.3	0.816	СНиП 2.01.07-85, tab. 2

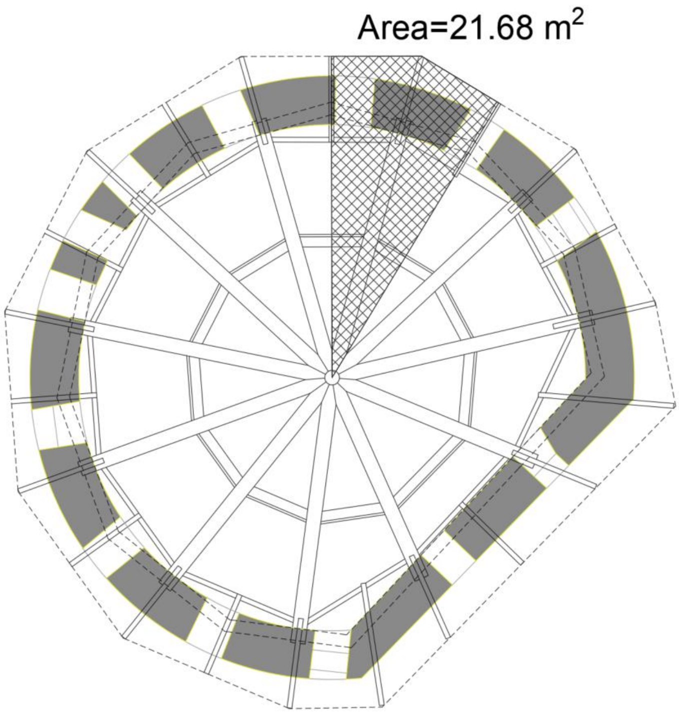

Figure 3 Load surface from wooden shingles

Snow load is computed with relation from [3]:

$$S = S_0 \cdot \mu$$

From annex 3 of СНиП 2.01.07-85. Нагрузки и воздействия the coefficient $\mu = 0$, thus makes the snow load on roof equals to 0.

2.3. Calculus

The design will be performed in FEM software - SCAD Soft. SCAD is an integrated system for finite element structural analysis and design. SCAD includes a highly developed library of finite elements for modeling bar, plate, solid and combined structures, modules of stability analysis, building design stress combinations, verifying stressed state of structural elements according to various failure theories, determining forces with which a fragment affects the whole structure, calculating forces and displacement caused by loading combinations.

3. RESULTS

3.1. Modeled structure

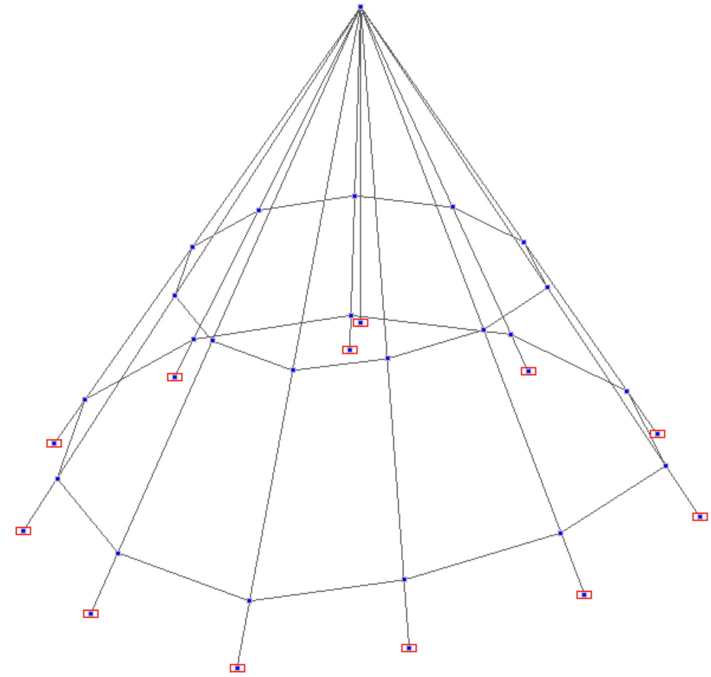

Figure 4 3D view of structure

Roof elements are design as bars that can be subjected to compression, tension, bending and torsion.

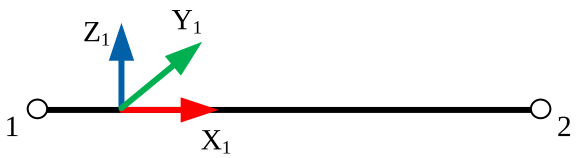

Figure 5 Generic bar in SCAD++

The element perceives the following types of efforts:

- N – axial force. The positive value corresponds to tension
- M_k – torsion moment of the X_1 axis. The positive value corresponds to the counterclockwise action of the moment, if you look at the end of the X_1 axis at the section that belongs to the bar.
- M_y – bending moment of the Y_1 axis. The positive value corresponds to the counterclockwise action of the moment, if you look at the end of the Y_1 axis at the section that belongs to the bar.
- M_Z – bending moment of the Z_1 axis. The positive value corresponds to the counterclockwise action of the moment, if you look at the end of the Z_1 axis at the section that belongs to the bar.
- Q_y – shear force along the Y_1 axis. The positive value corresponds to the coincidence of the force direction with the axis Y_1, for section that belongs to the end of the bar.
- Q_z – shear force along the Z_1 axis. The positive value corresponds to the coincidence of the force direction with the axis Z_1, for section that belongs to the end of the bar.

3.2. Dynamic proprieties of structure

Table 2 Dynamic proprieties of structure

Load case nr.		Mode	Frequencies		Period	Modal mass (%)		
		NR.	rad/sec	Hz	sec	X	Y	Z
3	Seismic action along X axis	1	7.951	1.265	0.79	0.015	17.377	1.014
		2	12.031	1.915	0.522	0	3.817	0.156
		3	13.222	2.104	0.475	0.014	0.053	0.865
		4	18.332	2.918	0.343	77.08	3.618	0.045
		5	19.574	3.115	0.321	0.602	0.384	0.054
		Sum of modal mass				77.711	25.249	2.134
4	Seismic action along Y axis	1	7.951	1.265	0.79	0.015	17.377	1.014
		2	12.031	1.915	0.522	0	3.817	0.156
		3	13.222	2.104	0.475	0.014	0.053	0.865
		4	18.332	2.918	0.343	77.08	3.618	0.045
		5	19.574	3.115	0.321	0.602	0.384	0.054
		Sum of modal mass				77.711	25.249	2.134

3.3. Efforts in elements of roof

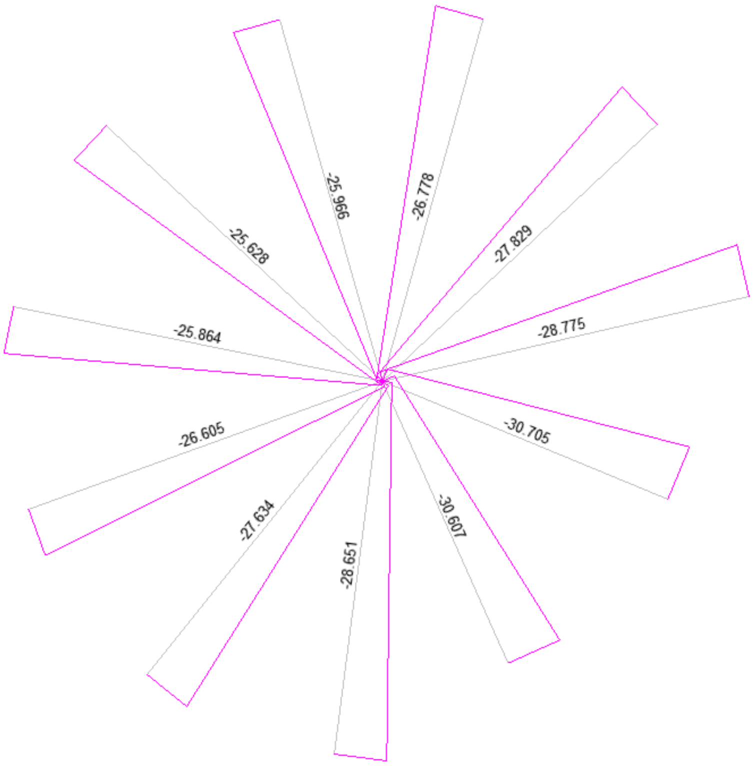

Figure 6 Axial forces in beams

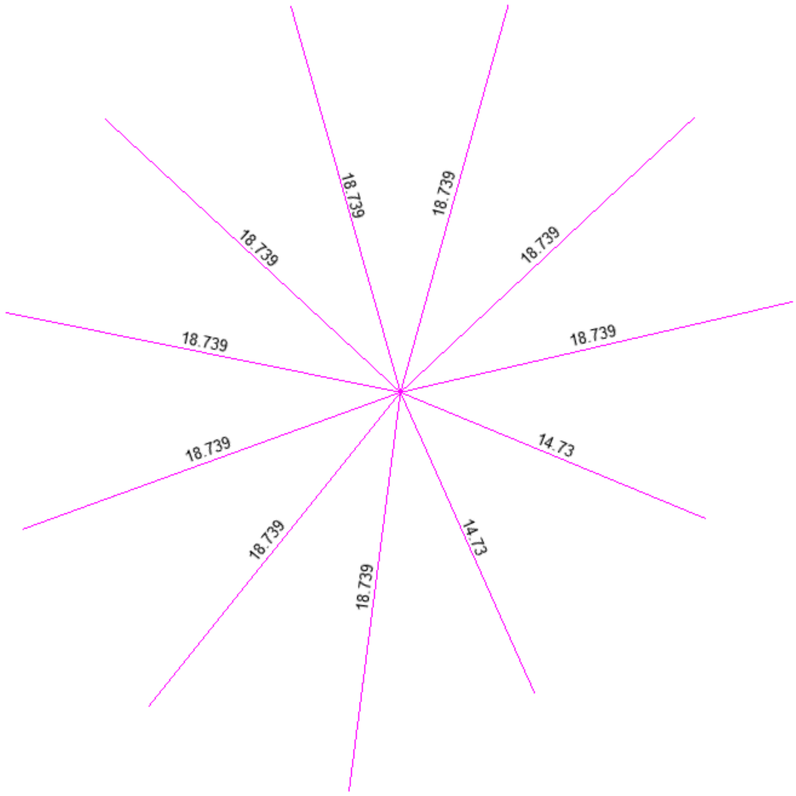

Figure 7 Bending moments in beams

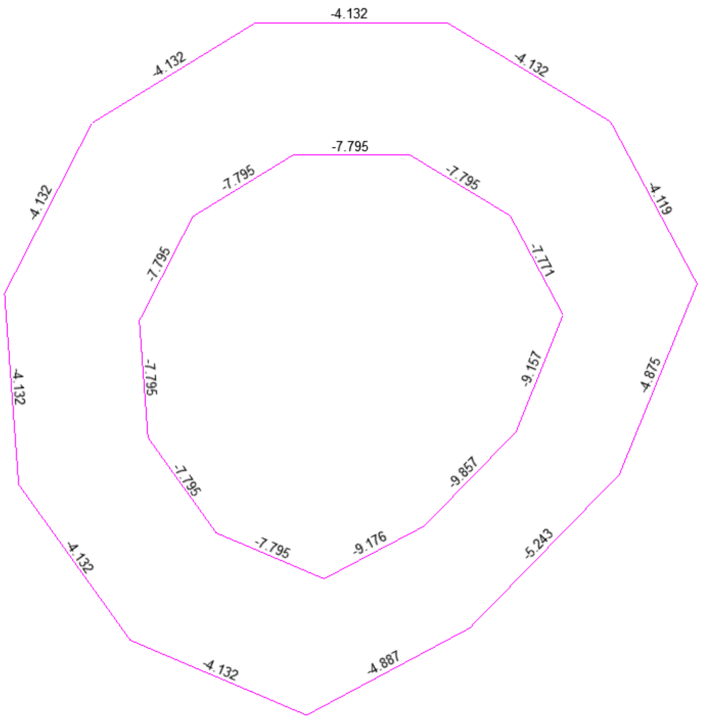

Figure 8 Axial forces in stiffening rings

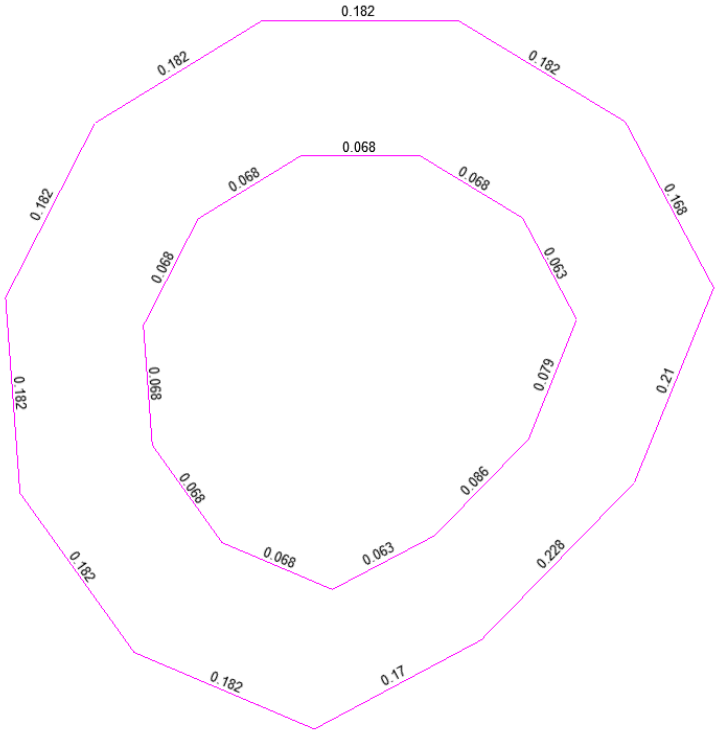

Figure 9 Bending moments in stiffening rings

For detailed information about each member see annex.

3.4. Verification of elements

For verification will be taken an element with the highest:

- Axial force and bending moment that corresponds to these axial force
- Bending moment and the axial force that corresponds to these axial force

Beam with the highest axial moment is beam nr. $B-1$:

$$N_{max} = -30.705\ (kN) \quad M_{coresp} = 14.73\ (kNm)$$

Beam with the highest bending moment is beam $B-11$:

$$M_{max} = 18.739\ (kNm) \ \ N_{coresp} = -28.775 (kN)$$

1st stiffening ring with the highest axial force is beam $R1-2$

$$N = -5.243\ (kN) \quad M_{coresp} = 0.228\ (kNm)$$

2nd stiffening ring with the highest axial force is beam $R1-2$

$$N = -9.857\ (kN) \quad M_{coresp} = 0.086\ (kNm)$$

Column $C-1$:

$$N_{max} = 21.3\ (kN)$$

3.4.1. Main beam

Verification of element subjected to combined stress (compression and bending) will be made according to NCM F.05.01-2007. „*Proiectarea construcțiilor din lemn*”.

$$\sigma_{max} = \gamma_n \left(\frac{N}{A_c} + \frac{M_d}{W_c}\right) \leq R_{c_{II}}$$

, where $M_d = \frac{M}{\xi}$

A_c – cross section of element

W_c – shear rigidity factor

ξ – coefficient which varies from 1 to 0 and takes into account the additional moment created by longitudinal force as a result of a bending of element and is computed by following expration:

$$\xi = 1 - \frac{N}{\varphi \cdot R_{cII} \cdot A_c}$$

, where φ – coefficient for any flexibility of the element. May be greater than 1 and is computed with following expression:

$$\varphi = \frac{3000}{\lambda^2}$$

$$\lambda = \frac{l_{ef}}{i_{x(y)}}$$

, where λ – is flexibility of element

l_{ef} – design length equal to $l_{ef} = l \cdot \mu$. In studied case $\mu = 1$

$i_{x(y)}$ – inertia radius

Table 3 Efforts in beam B-1 and B-11

Element	$N\ (kN)$	$M\ (kNm)$	ξ	$M_d\ (kNm)$	$\sigma_{max}\ (MPa)$	$R_{cII}\ (MPa)$
$B-1$	-30.705	14.73	0.862	17.091	4.139	10.4
$B-11$	-28.775	18.739	0.871	21.525	5.103	10.4

Also, should be verified condition $f < f_i$. Where $f_i = \frac{l}{150} = 72.6\ (mm)$. As can be noticed from annex the maximum displacement along Z axis is -4.098 mm, so serviceability condition is satisfied.

3.4.2. 1st stiffening ring

As can be noticed, the bending moment is small comparatively to axial forces, thus bending moment can be neglected and considered that beam is subjected to compression. The verification should be made for:

- Resistance

$$\sigma = \frac{N\gamma_n}{A} \leq R_{cII}$$

$$\sigma = \frac{5.243 \cdot 1}{0.02} = 0.262\ (MPa) < 10.4\ (MPa)$$

Condition is satisfied.

- Stability

$$\sigma = \frac{N\gamma_n}{\varphi_c A} \leq R_{cII}$$

$$\lambda = \frac{l_{ef}}{i_{min}} = \frac{2.99\ m}{27.7\ mm} = 104.18$$

The coefficient φ for $\lambda > 70$ is equal to:

$$\varphi = \frac{3000}{\lambda^2} = 0.275$$

$$\sigma = \frac{5.243 \cdot 1}{0.09 \cdot 0.275} = 0.953\ (MPa) < 10.4\ (MPa)$$

, where $A = b \cdot h = 100 \cdot 200 = 20000\ mm^2 = 0.02\ m^2$

Condition is satisfied.

3.4.3. 2nd stiffening ring

As can be noticed, the bending moment is small comparatively to axial forces, thus bending moment can be neglected and considered that beam is subjected to compression. The verification should be made for:

- Resistance

$$\sigma = \frac{N\gamma_n}{A} \leq R_{cII}$$

$$\sigma = \frac{9.871 \cdot 1}{0.02} = 0.058\ (MPa) < 10.4\ (MPa)$$

Condition is satisfied.

- Stability

$$\sigma = \frac{N\gamma_n}{\varphi_c A} \leq R_{cII}$$

$$\lambda = \frac{l_{ef}}{i_{min}} = \frac{1.83\ m}{27.7\ mm} = 63.76$$

The coefficient φ for $\lambda \leq 70$ is equal to:

$$\varphi = 1 - 0.8 \cdot \left(\frac{\lambda}{100}\right)^2 = 0.675$$

$$\sigma = \frac{9.871 \cdot 1}{0.02 \cdot 0.675} = 0.731\ (MPa) < 10.4\ (MPa)$$

Condition is satisfied.

3.4.4. Column C-1

The verification should be made for:

- Resistance

$$\sigma = \frac{N\gamma_n}{A} \leq R_{cII}$$

$$\sigma = \frac{21.3 \cdot 1}{0.071} = 0.3\ (MPa) < 10.4\ (MPa)$$

Condition is satisfied.

- Stability

$$\sigma = \frac{N\gamma_n}{\varphi_c A} \leq R_{cII}$$

$$\lambda = \frac{l_{ef}}{i_{min}} = \frac{5.58\ m}{75\ mm} = 74.4$$

The coefficient φ for $\lambda > 70$ is equal to:

$$\varphi = \frac{3000}{\lambda^2} = 0.542$$

$$\sigma = \frac{21.3 \cdot 1}{0.071 \cdot 0.542} = 0.554\ (MPa) < 10.4\ (MPa)$$

, where $A = \pi R^2 = 3.14 \cdot 150^2\ mm = 70650\ mm^2 = 0.071\ m^2$

Condition is satisfied.

4. CONCLUSION

The structural analysis of new roof structure was made. The following conclusion can be stated:

1. All elements and loads were modeled in FEM software SCAD++. Beam elements were modeled as bars with given rigidity and section proprieties as showing in Annex 1. Wooden shingles were modeled as load.
2. Because of large angle, snow load was considered to be equal to zero.
3. As can be seen from analysis, all efforts are evenly distributed along beams. Small discrepancy can be noticed in beams $B-1$ and $B-2$. These is caused due to the angle at which theses beams are placed relative to other beams.
4. The analysis of roof elements pointed out that main beams are subjected to a combined stress (compression and bending), and stiffening rings are subjected to compression.
5. From analysis of the most subjected beams, one could notice that they pass verification, i.e.:

$$\sigma_{max} < R_{cII}$$

6. From the verification could be pointed out that stiffening rings have a resistance reserve more than 80 %.
7. The structural resistance and stability of all structure it-self is ensured.

ANNEX 1 Section proprieties

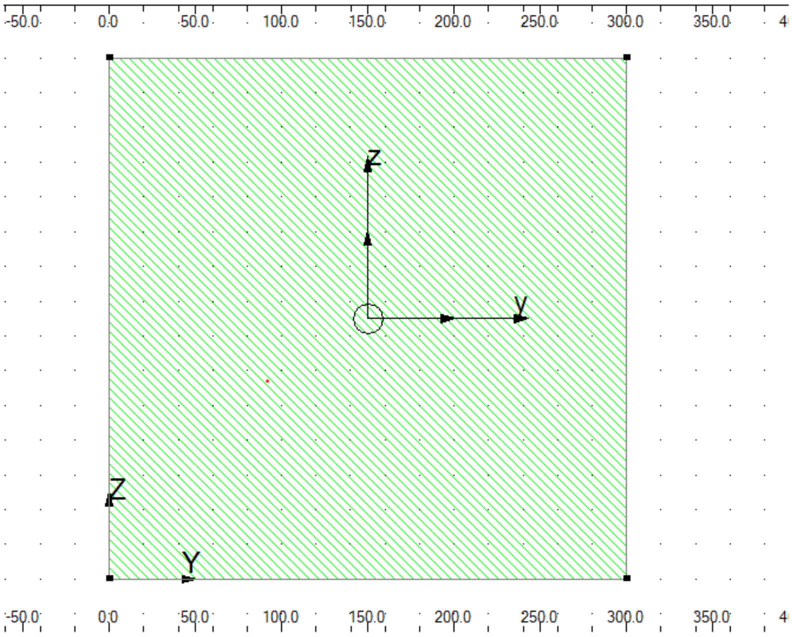

Figure 10 Beam section 300 x 300

Geometry description

Point no.	Y	Z
1	0.00 mm	0.00 mm
2	300.00 mm	0.00 mm
3	300.00 mm	300.00 mm
4	0.00 mm	300.00 mm

General results

Area

A = 900.000 cm2

Center of gravity

Yc = 150.00 mm
Zc = 150.00 mm

Perimeter

S = 1200.00 mm

Principal system

Angle

alpha = 0.0 Deg

Moments of inertia

Ix = 113945.726 cm4
Iy = 67500.000 cm4
Iz = 67500.000 cm4

Radii of inertia

iy = 86.60 mm
iz = 86.60 mm

Shear areas

Ay = 750.000 cm2
Az = 750.000 cm2

Elastic section moduli

Wely	=	4500.000 cm3
Welz	=	4500.000 cm3

Shear rigidity factors

Wy	=	600.000 cm2
Wz	=	600.000 cm2

Plastic section moduli

Wply	=	6750.000 cm3
Wplz	=	6750.000 cm3

Maximum distances

Vy	=	150.00 mm
Vpy	=	150.00 mm
Vz	=	150.00 mm
Vpz	=	150.00 mm

Central system

Moments of inertia

Iyc	=	67500.000 cm4
Izc	=	67500.000 cm4
Iyczc	=	0.000 cm4

Radii of inertia

iyc	=	86.60 mm
izc	=	86.60 mm

Maximum distances

Vyc	=	150.00 mm
Vpyc	=	150.00 mm
Vzc	=	150.00 mm
Vpzc	=	150.00 mm

Arbitrary system

System position

yc'	=	150.00 mm	Angle =	0.0 Deg
zc'	=	150.00 mm		

Moments of inertia

Iy'	=	67500.000 cm4
Iz'	=	67500.000 cm4
Iy'z'	=	0.000 cm4

Radii of inertia

iyc	=	86.60 mm
izc	=	86.60 mm

First moments of area

Sy'	=	0.000 cm3
Sz'	=	0.000 cm3

Maximum distances

Vy'	=	150.00 mm
Vpy'	=	150.00 mm
Vz'	=	150.00 mm
Vpz'	=	150.00 mm

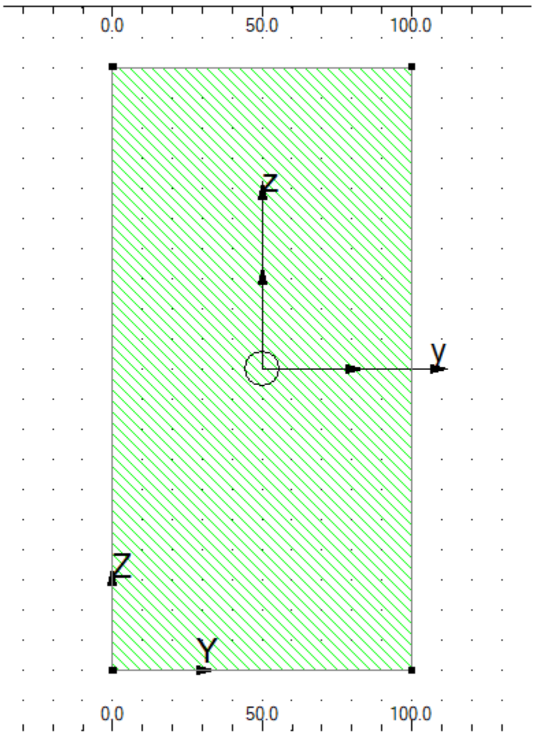

Figure 11 Beam section 100 x 200

Geometry description

Point no.	Y	Z
1	0.00 mm	0.00 mm
2	100.00 mm	0.00 mm
3	100.00 mm	200.00 mm
4	0.00 mm	200.00 mm

General results

Area

A = 200.000 cm2

Center of gravity

Yc = 50.00 mm
Zc = 100.00 mm

Perimeter

S = 600.00 mm

Principal system

Angle

alpha = 0.0 Deg

Moments of inertia

Ix = 4579.061 cm4
Iy = 6666.667 cm4
Iz = 1666.667 cm4

Radii of inertia

iy = 57.74 mm
iz = 28.87 mm

Shear areas

Ay = 166.667 cm2

	Az = 166.667 cm2
Elastic section moduli	
	Wely = 666.667 cm3
	Welz = 333.333 cm3
Shear rigidity factors	
	Wy = 133.333 cm2
	Wz = 133.333 cm2
Plastic section moduli	
	Wply = 1000.000 cm3
	Wplz = 500.000 cm3
Maximum distances	
	Vy = 50.00 mm
	Vpy = 50.00 mm
	Vz = 100.00 mm
	Vpz = 100.00 mm

Central system

Moments of inertia	
	Iyc = 6666.667 cm4
	Izc = 1666.667 cm4
	Iyczc = -0.000 cm4
Radii of inertia	
	iyc = 57.74 mm
	izc = 28.87 mm
Maximum distances	
	Vyc = 50.00 mm
	Vpyc = 50.00 mm
	Vzc = 100.00 mm
	Vpzc = 100.00 mm

Arbitrary system

System position		
	yc' = 50.00 mm	Angle = 0.0 Deg
	zc' = 100.00 mm	
Moments of inertia		
	Iy' = 6666.667 cm4	
	Iz' = 1666.667 cm4	
	Iy'z' = 0.000 cm4	
Radii of inertia		
	iyc = 57.74 mm	
	izc = 28.87 mm	
First moments of area		
	Sy' = -0.000 cm3	
	Sz' = -0.000 cm3	
Maximum distances		
	Vy' = 50.00 mm	
	Vpy' = 50.00 mm	
	Vz' = 100.00 mm	
	Vpz' = 100.00 mm	

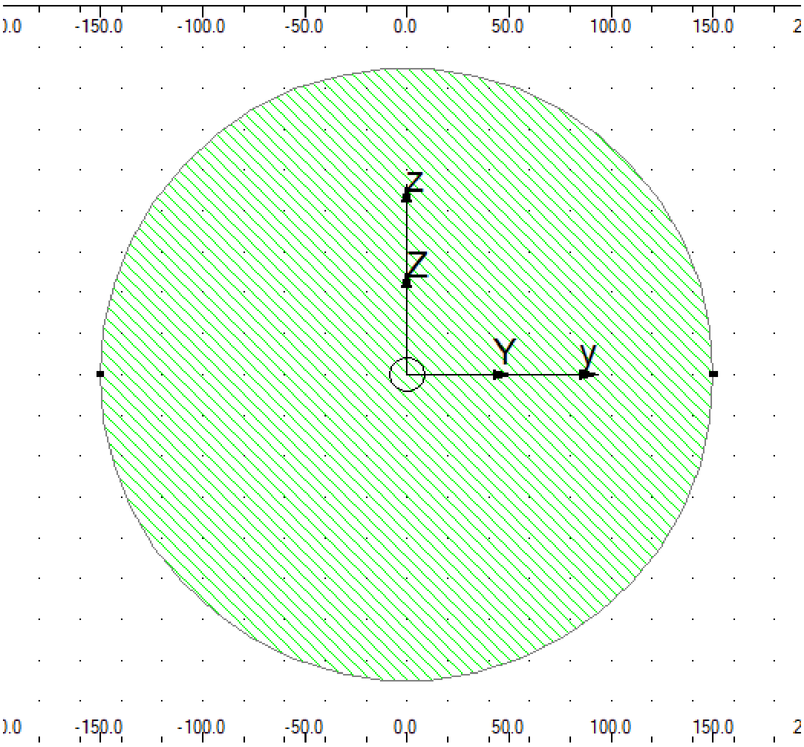

Figure 12 Column section Ø300

Geometry description

Point no.	Y	Z	
1	-150.00 mm	0.00 mm	Angle = 180.0 Deg
2	150.00 mm	0.00 mm	Angle = 180.0 Deg

General results

Area

A = 706.858 cm2

Center of gravity

Yc = 0.00 mm
Zc = 0.00 mm

Perimeter

S = 942.48 mm

Principal system

Angle

alpha = 0.0 Deg

Moments of inertia

Ix = 78860.395 cm4
Iy = 39760.782 cm4
Iz = 39760.782 cm4

Radii of inertia

iy = 75.00 mm
iz = 75.00 mm

Shear areas

Ay = 636.173 cm2
Az = 636.172 cm2

Elastic section moduli

Wely = 2650.719 cm3
Welz = 2650.719 cm3

Shear rigidity factors

Wy = 530.144 cm2
Wz = 530.144 cm2

Plastic section moduli

Wply = 4500.303 cm3
Wplz = 4500.303 cm3

Maximum distances

Vy = 150.00 mm
Vpy = 150.00 mm
Vz = 150.00 mm
Vpz = 150.00 mm

Central system

Moments of inertia

Iyc = 39760.782 cm4
Izc = 39760.782 cm4
Iyczc = -0.000 cm4

Radii of inertia

iyc = 75.00 mm
izc = 75.00 mm

Maximum distances

Vyc = 150.00 mm
Vpyc = 150.00 mm
Vzc = 150.00 mm
Vpzc = 150.00 mm

Arbitrary system

System position

yc' = 0.00 mm Angle = 0.0 Deg
zc' = 0.00 mm

Moments of inertia

Iy' = 39760.782 cm4
Iz' = 39760.782 cm4
Iy'z' = -0.000 cm4

Radii of inertia

iyc = 75.00 mm
izc = 75.00 mm

First moments of area

Sy' = 0.000 cm3
Sz' = 0.000 cm3

Maximum distances

Vy' = 150.00 mm
Vpy' = 150.00 mm
Vz' = 150.00 mm
Vpz' = 150.00 mm

ANNEX 2 Efforts in main beam

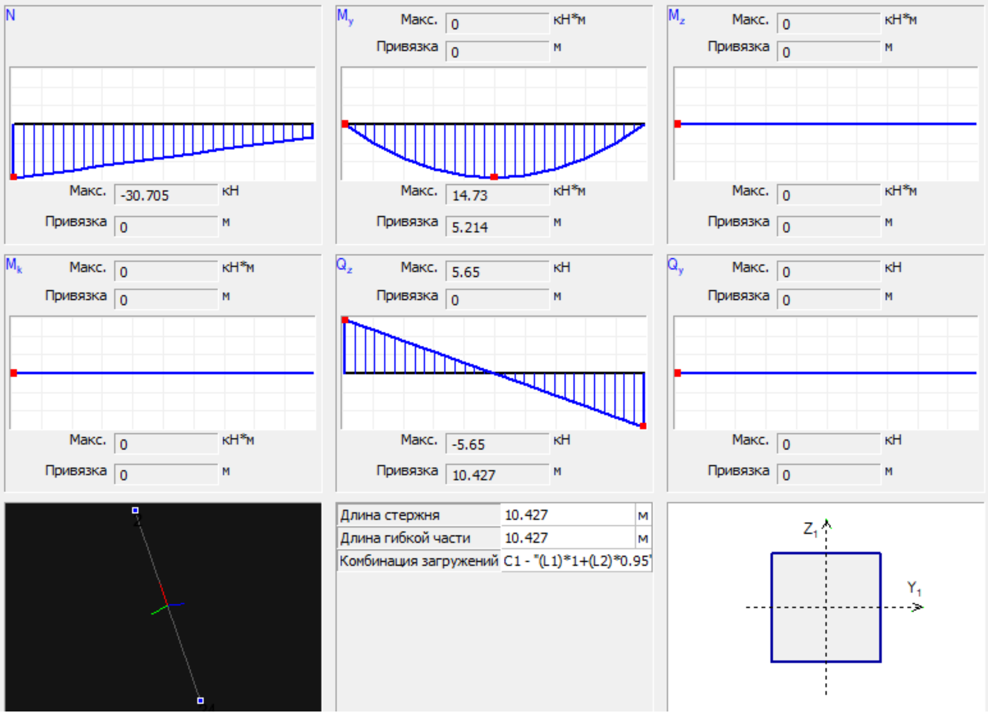

Figure 13 Beam B-1

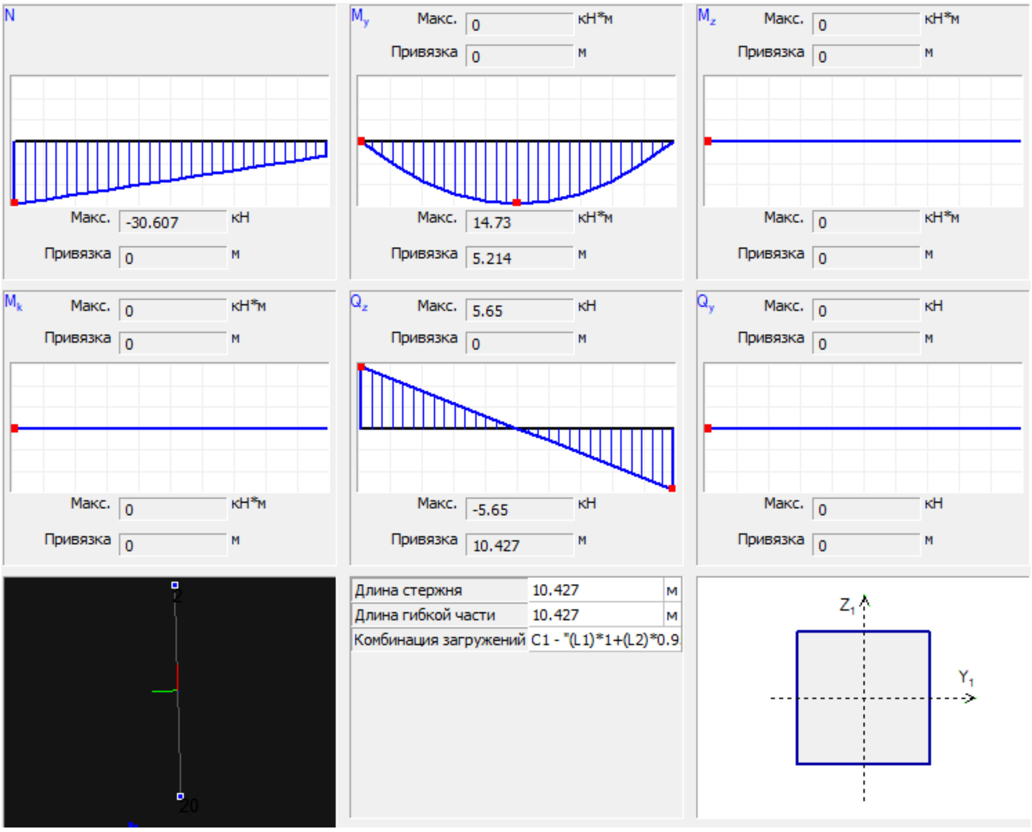

Figure 14 Beam B-2

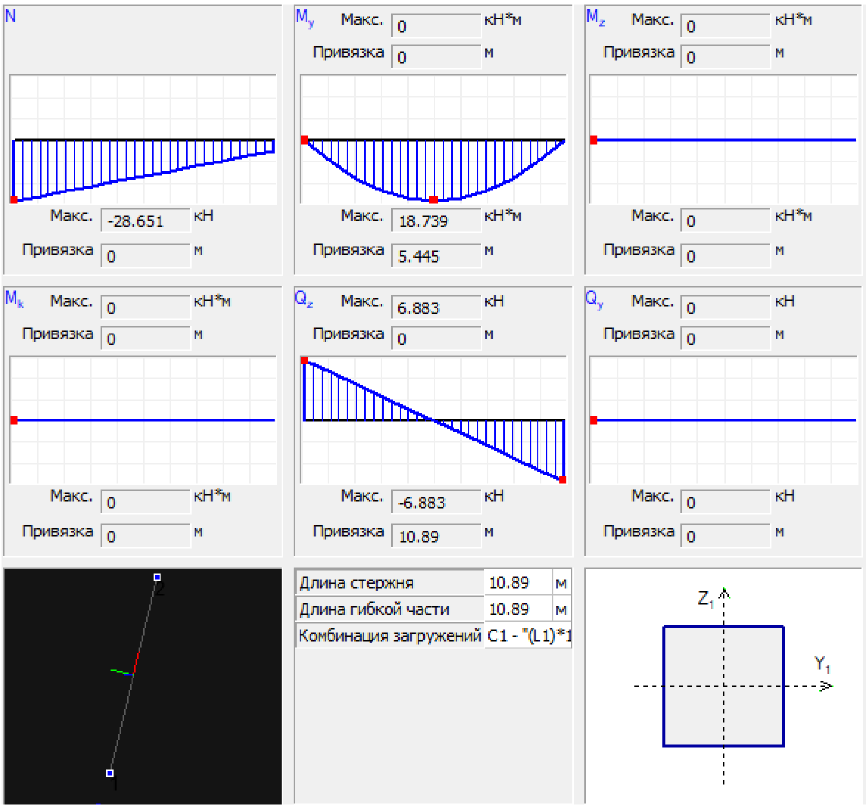

Figure 15 Beam B-3

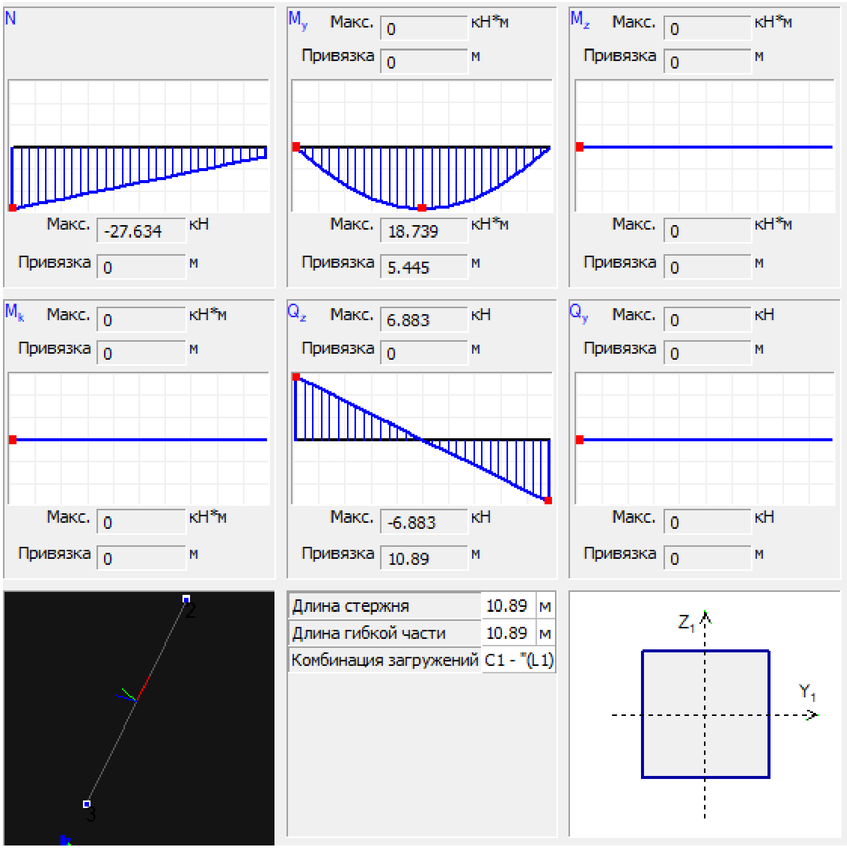

Figure 16 Bea, B-4

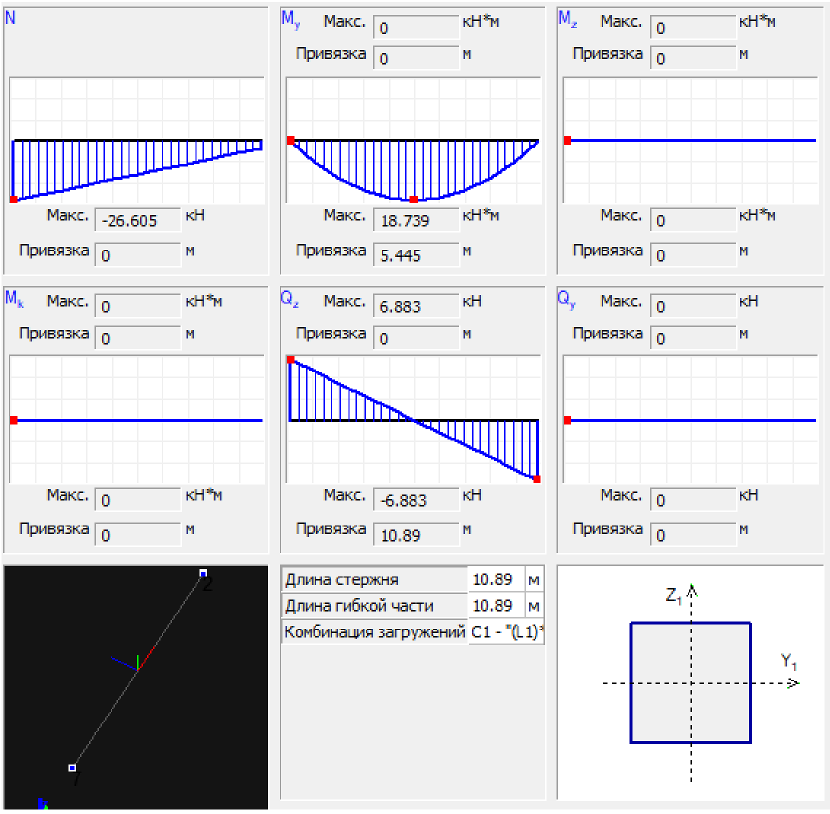

Figure 17 Beam B-5

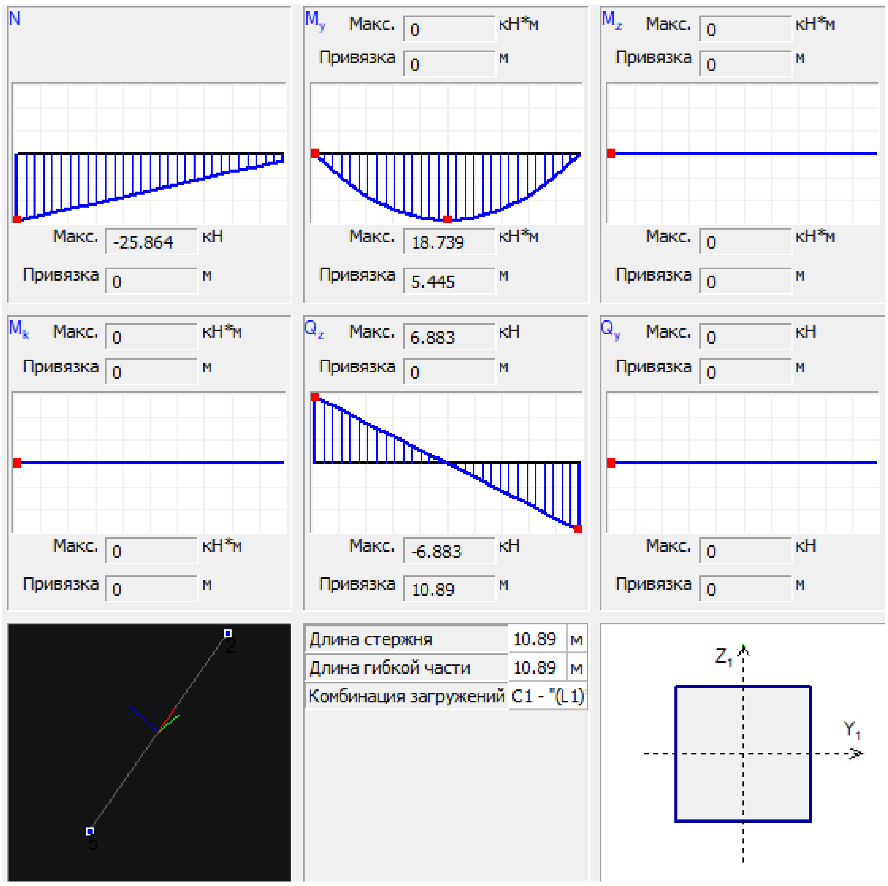

Figure 18 Beam B-6

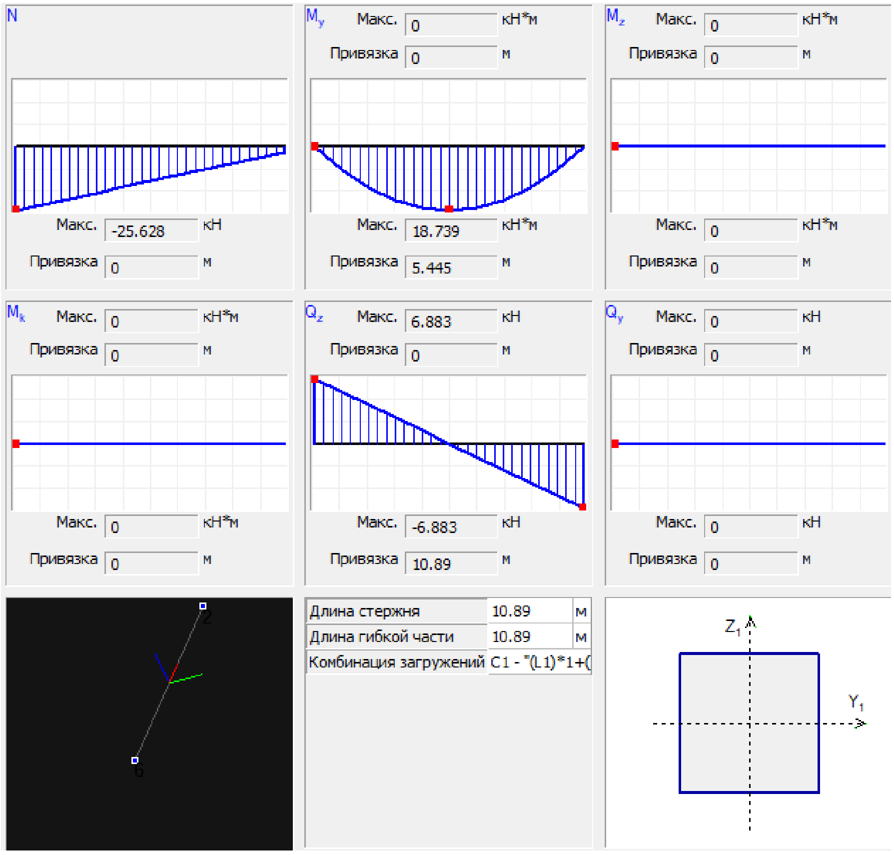

Figure 19 Beam B-7

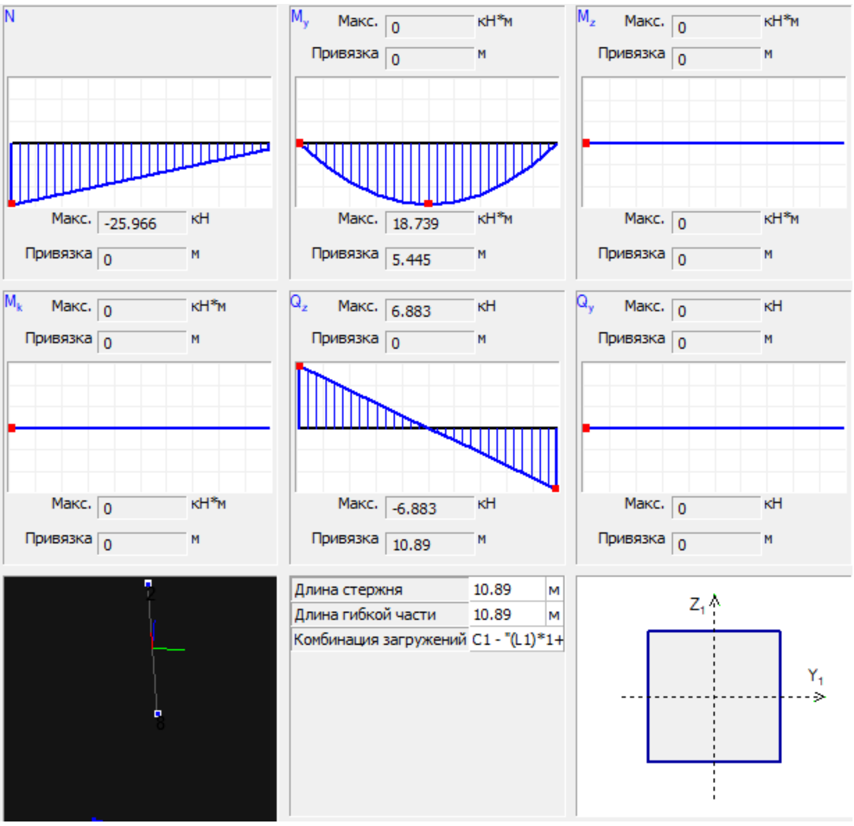

Figure 20 Beam B-8

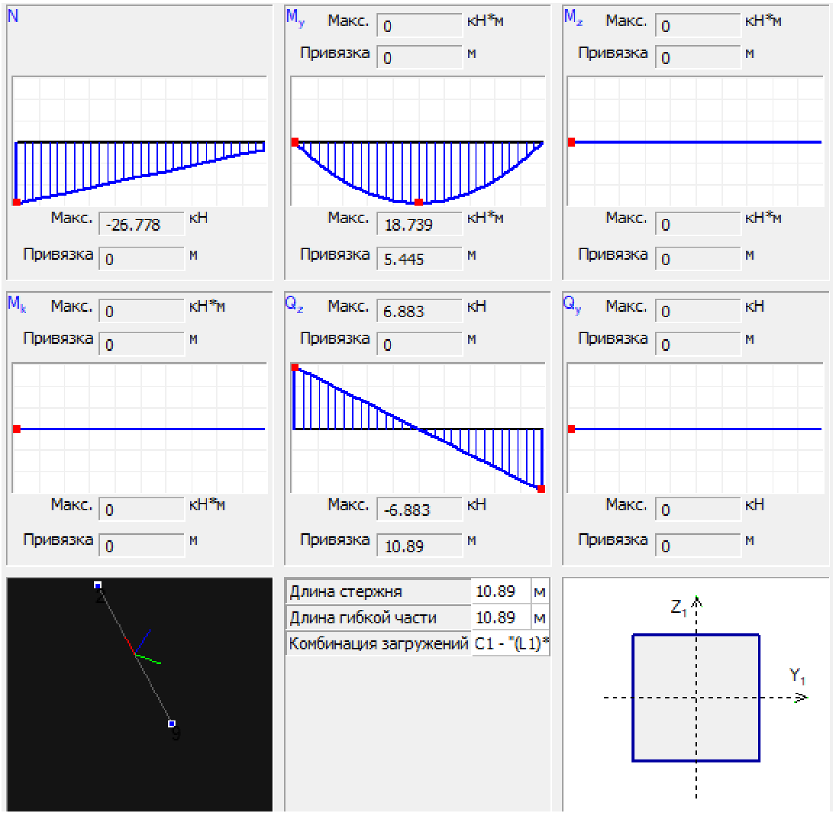

Figure 21 Beam B-9

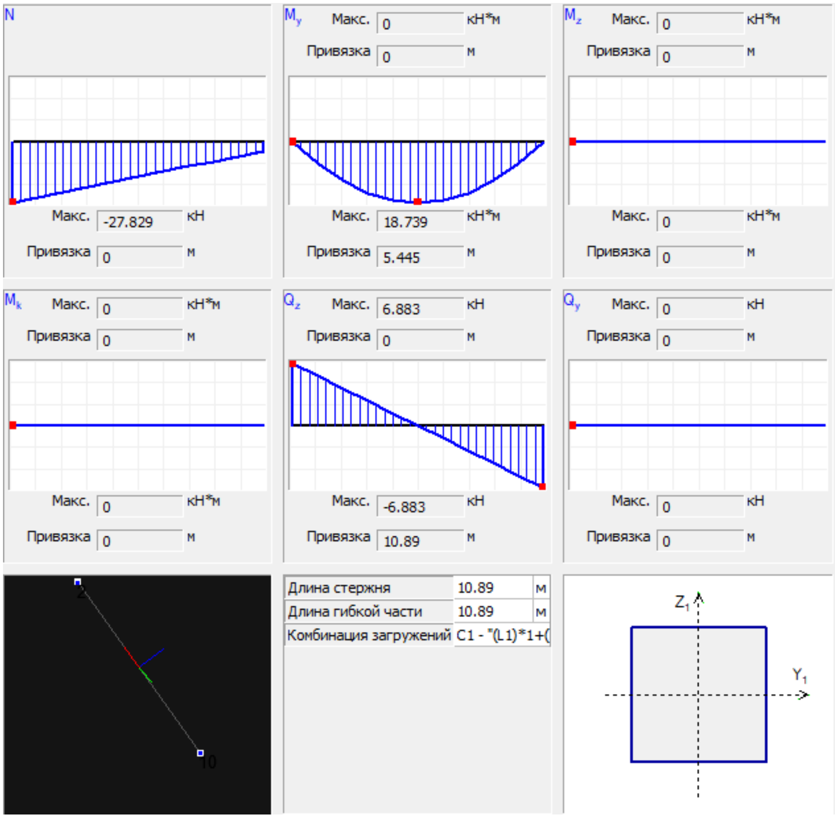

Figure 22 Beam B-10

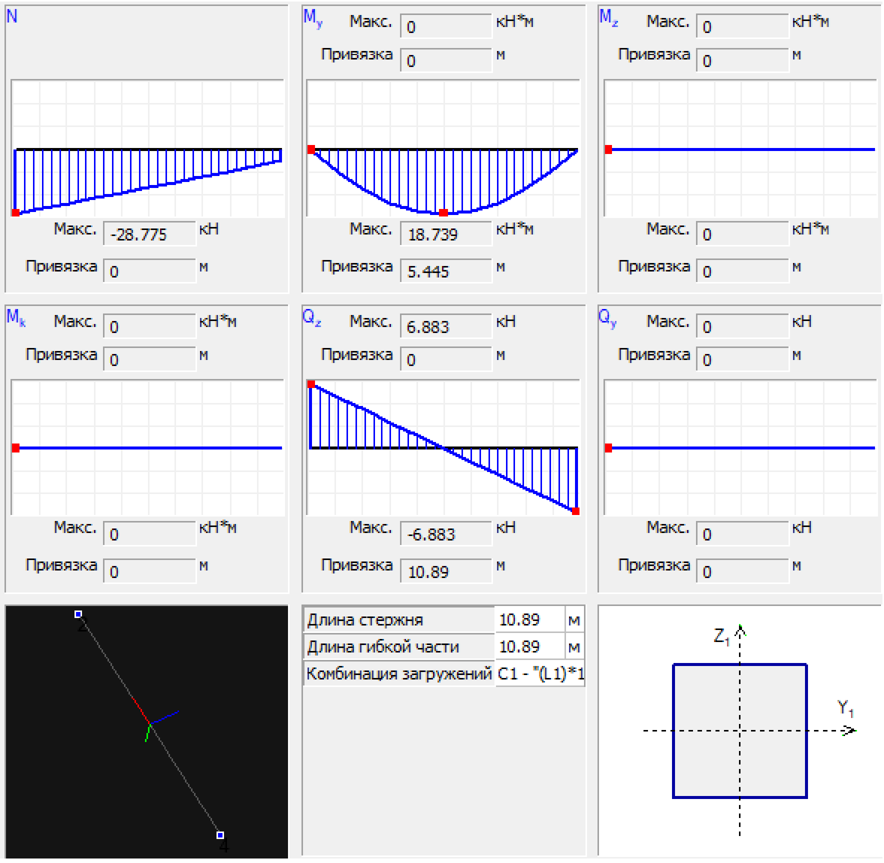

Figure 23 Beam B-11

ANNEX 3 Efforts in 1st stiffening ring

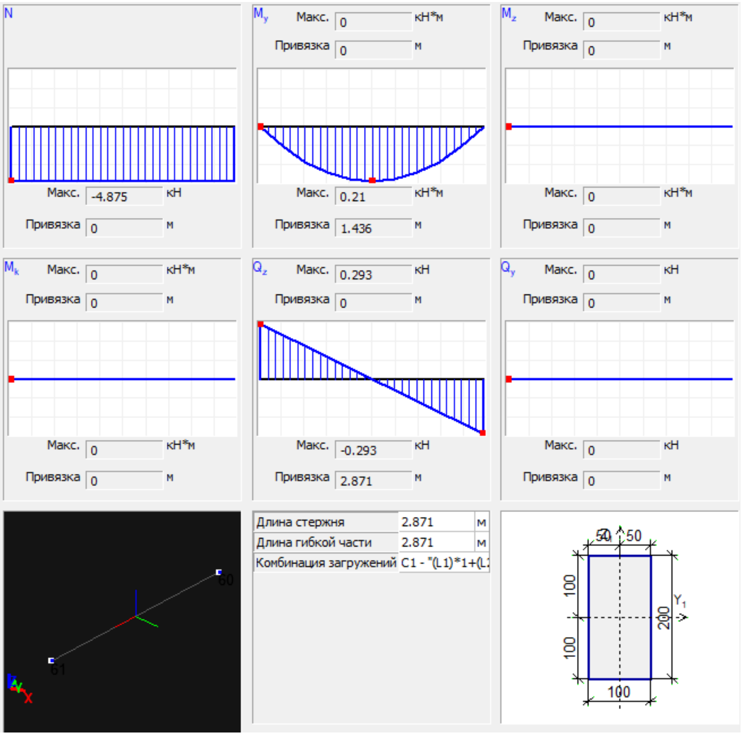

Figure 24 Beam R1-1

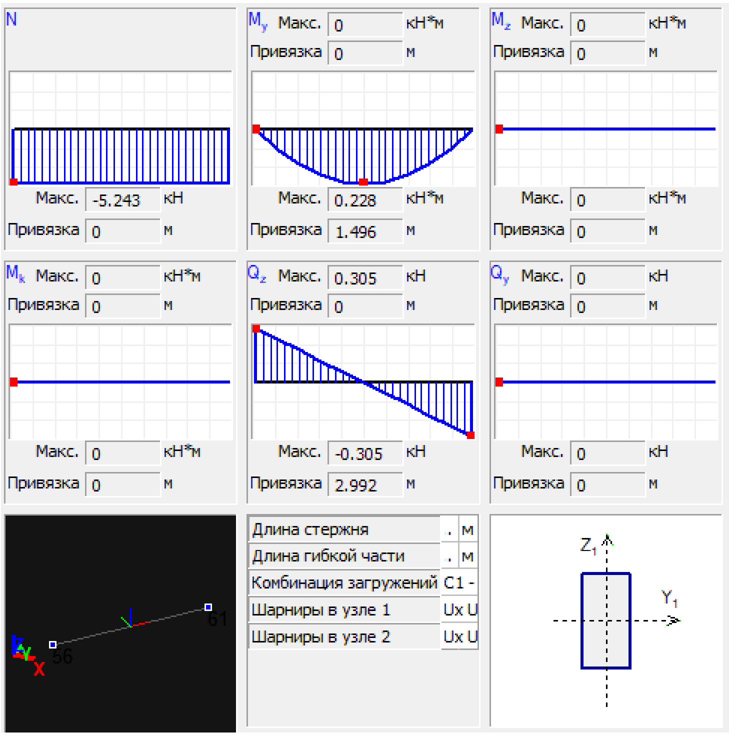

Figure 25 Beam R1-2

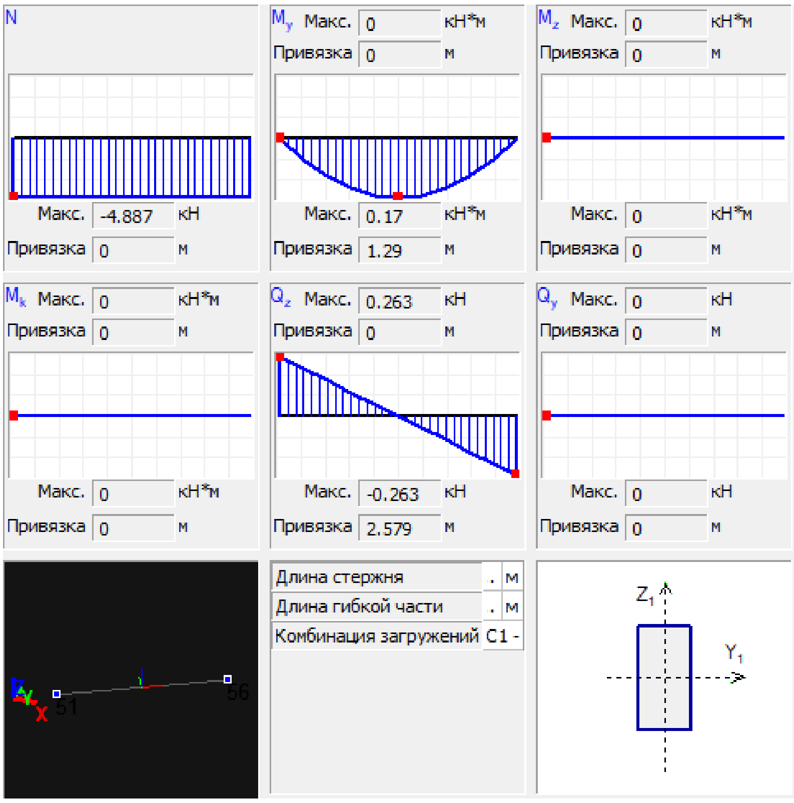

Figure 26 Beam R1-3

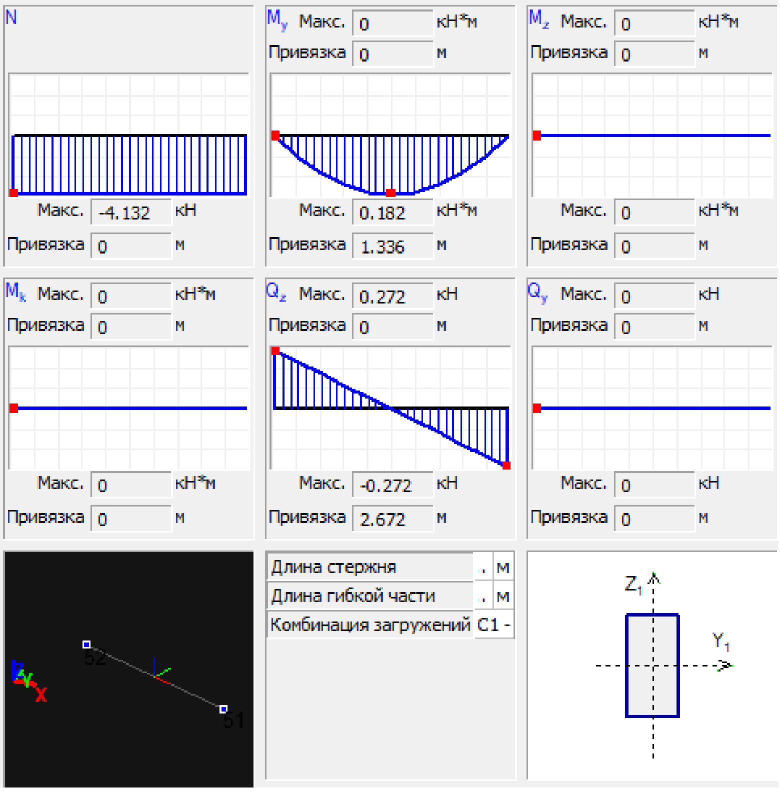

Figure 27 Beam R1-4

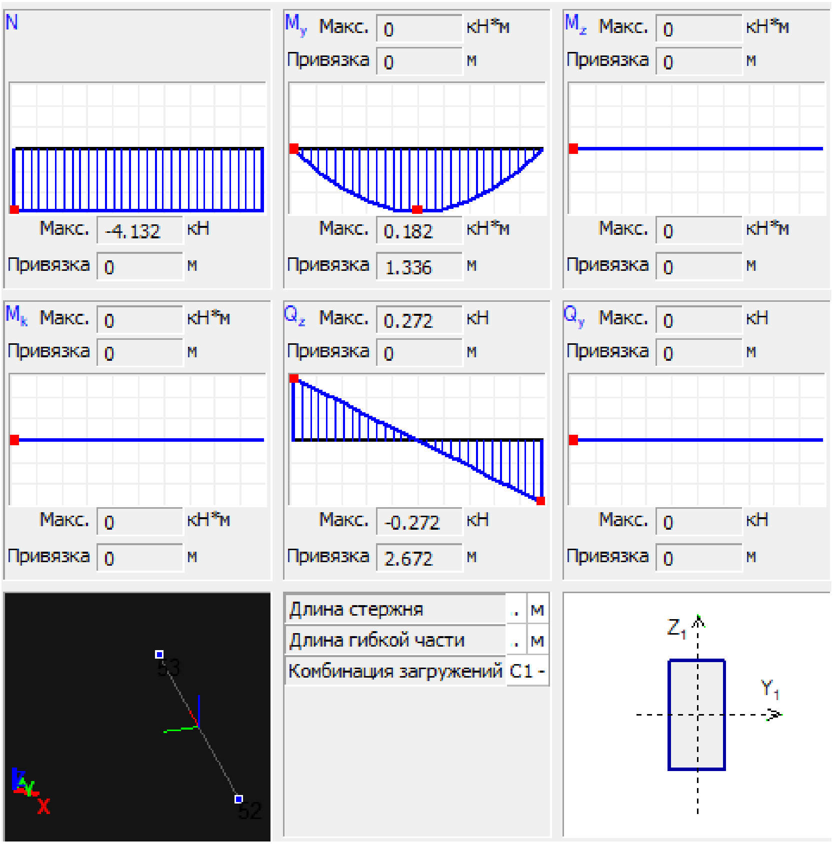

Figure 28 Beam R1-5, R1-6, R1-7, R1-8, R1-9, R1-10

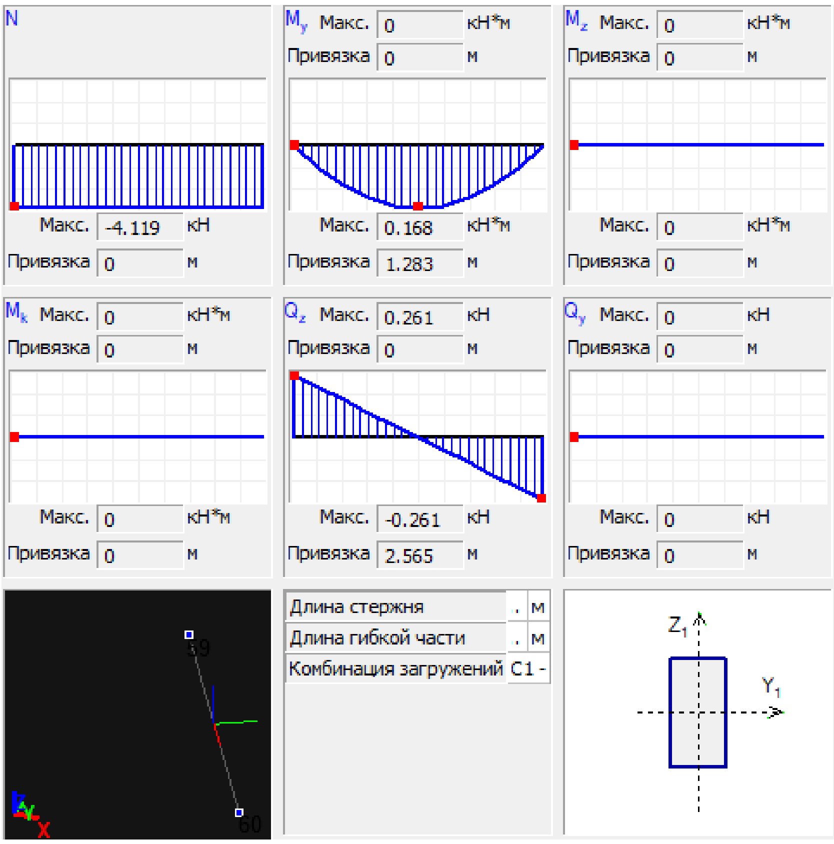

Figure 29 Beam R1-11

ANNEX 4 Efforts in 2nd stiffening ring

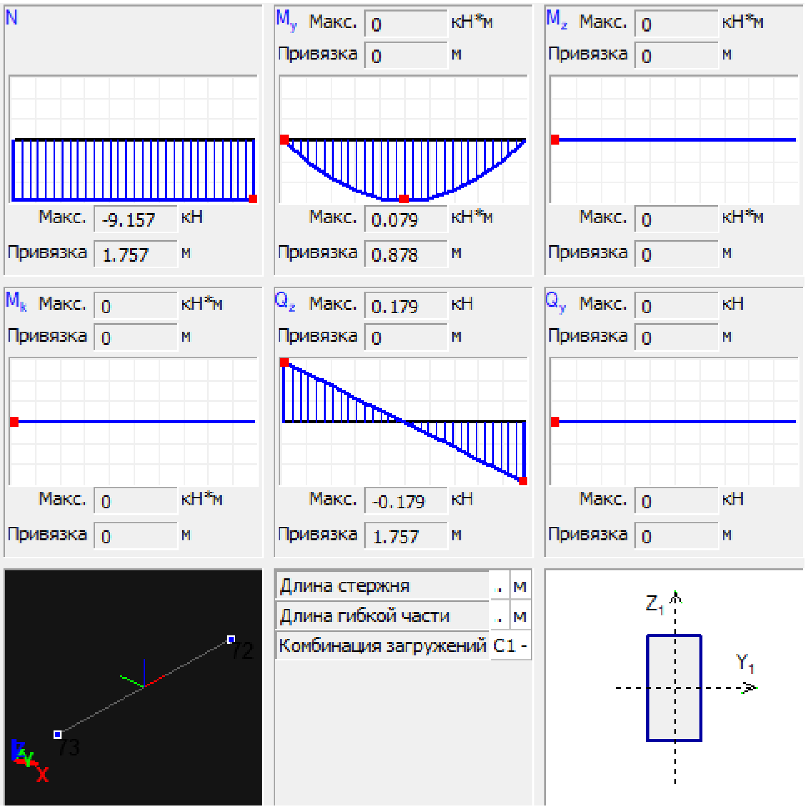

Figure 30 Beam R2-1

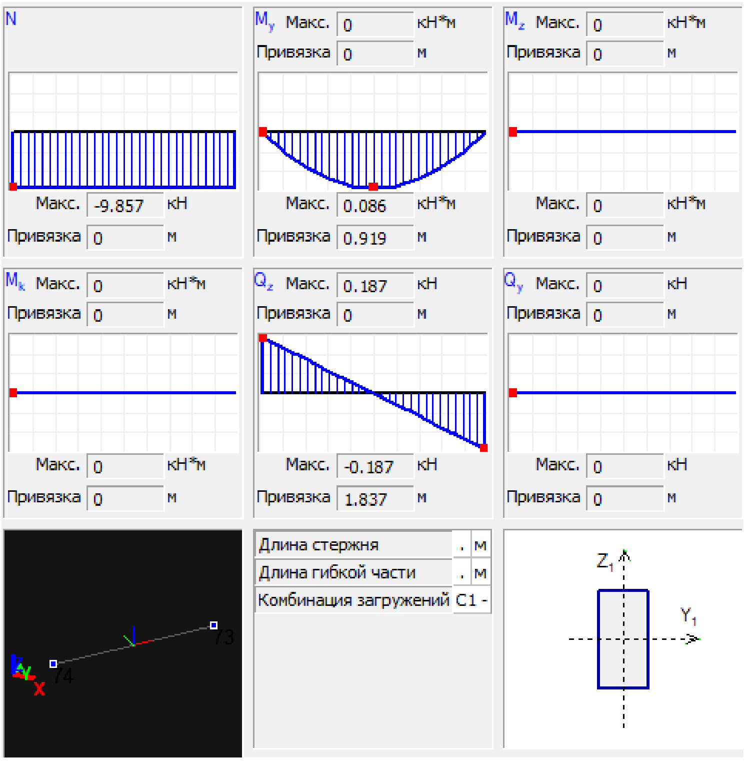

Figure 31 Beam R2-2

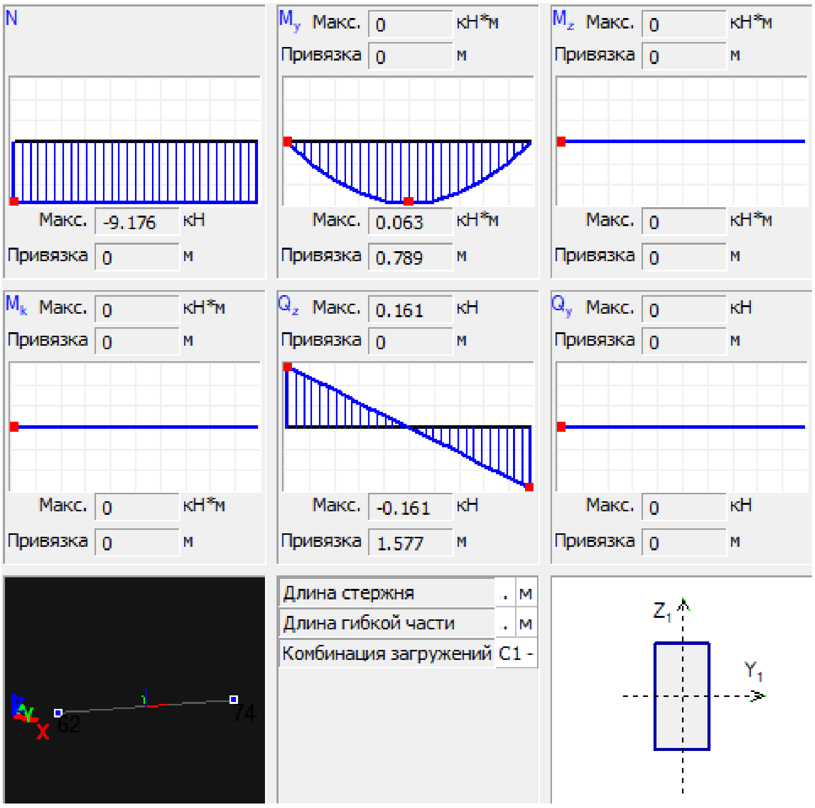

Figure 32 Beam R2-3

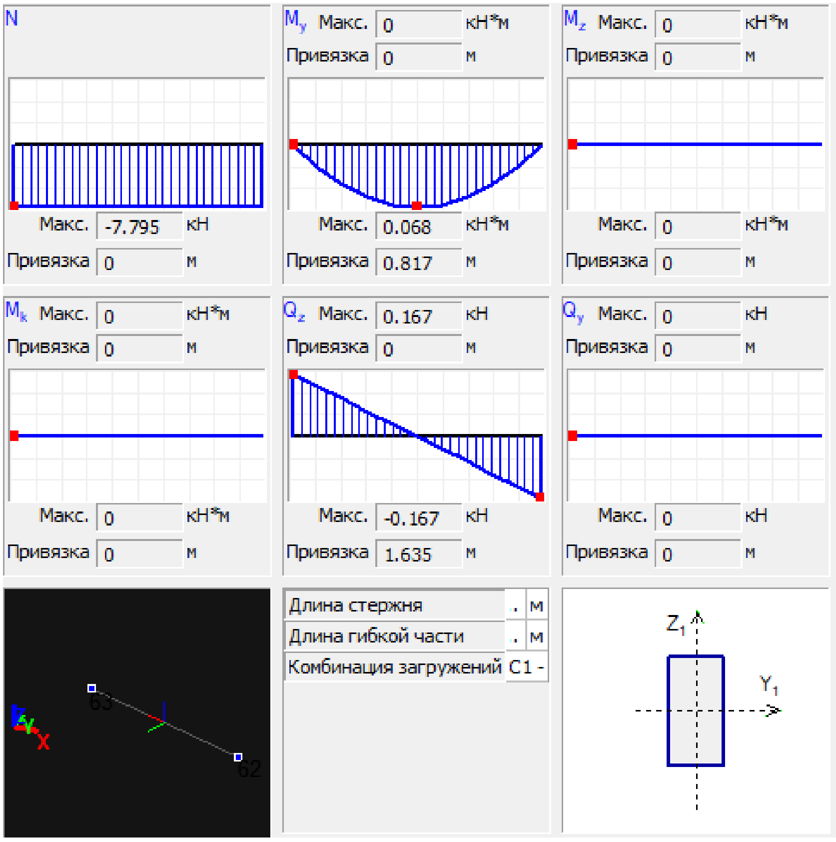

Figure 33 Beam R2-4, R2-5, R2-6, R2-7, R2-8, R2-9, R2-10

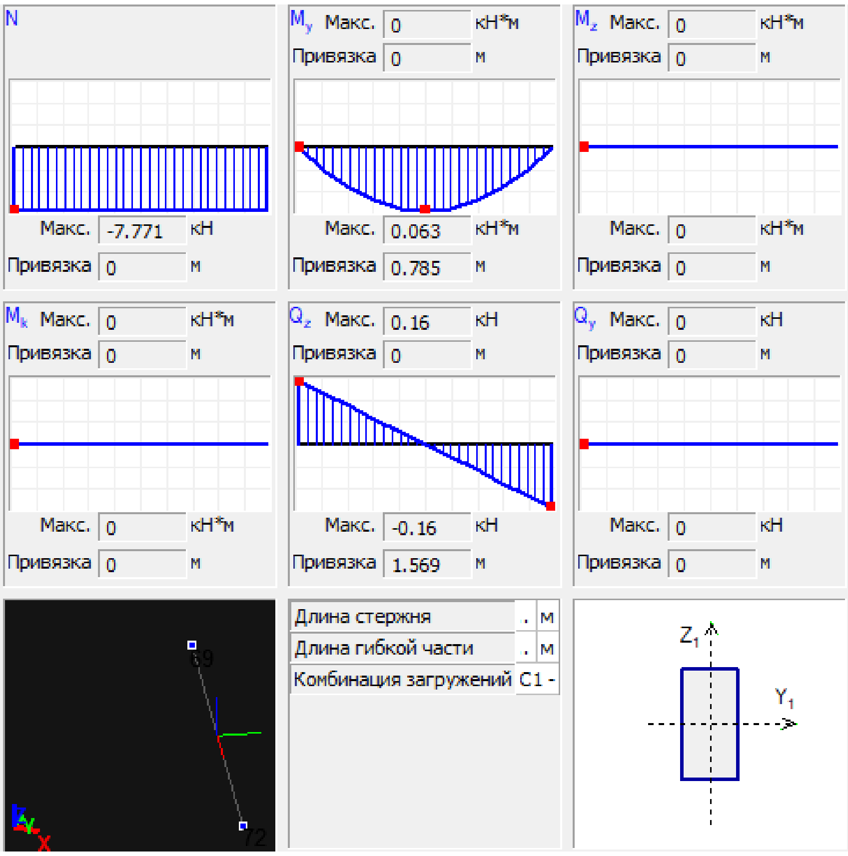

Figure 34 Beam R2-11

ANNEX 4 Structure displacements

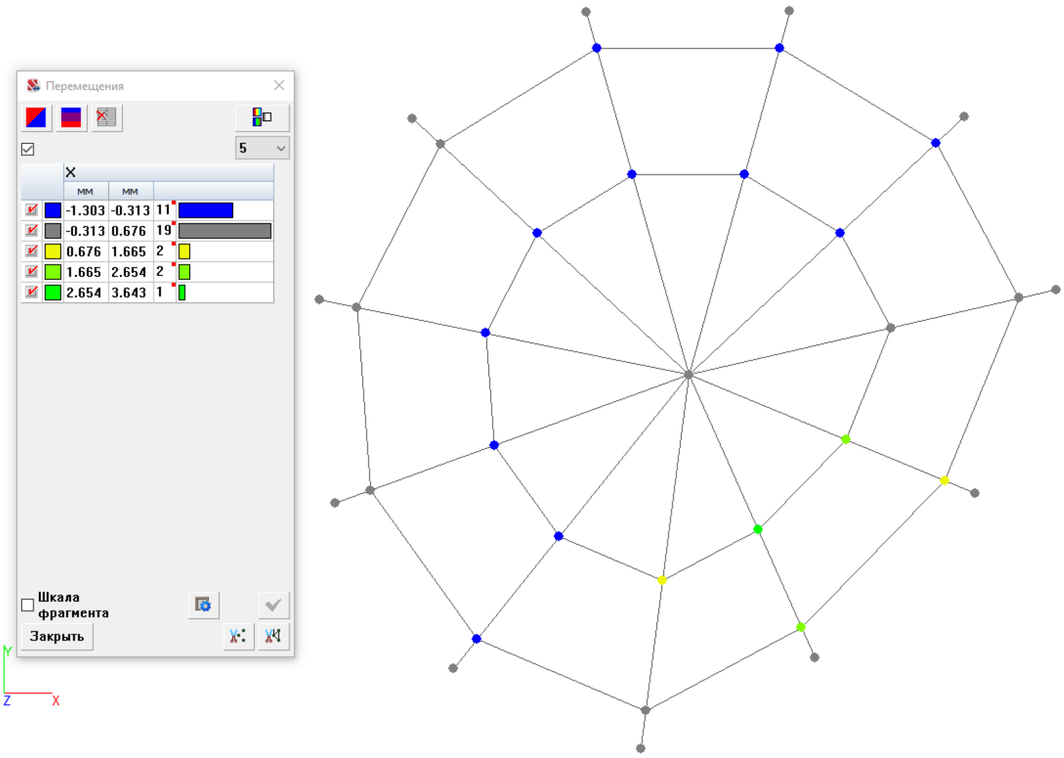

Figure 35 Displacements along X axis

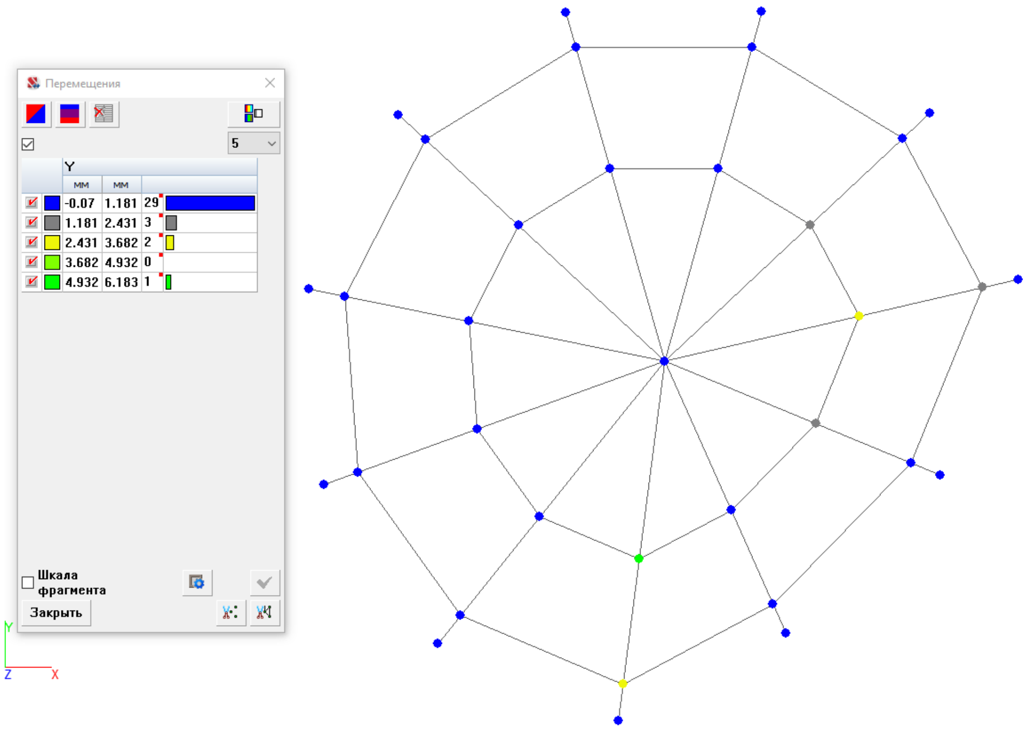

Figure 36 Displacements along Y axis

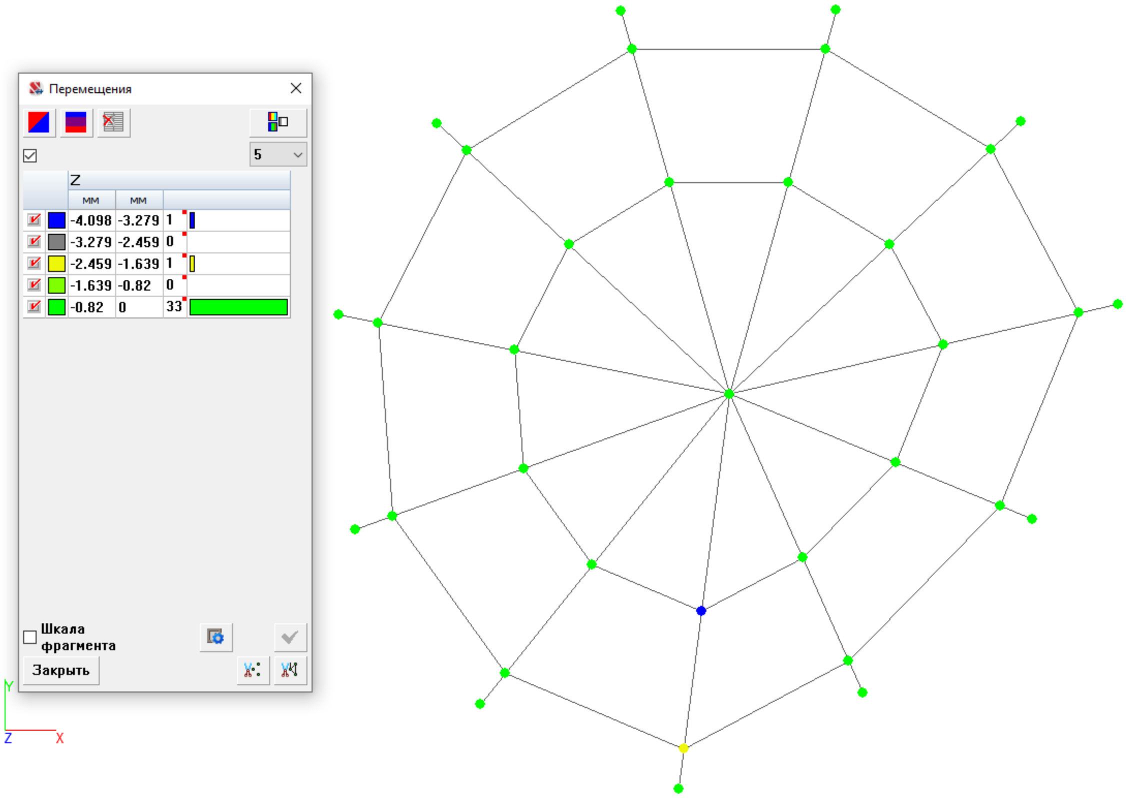

Figure 37 Displacements along Z axis

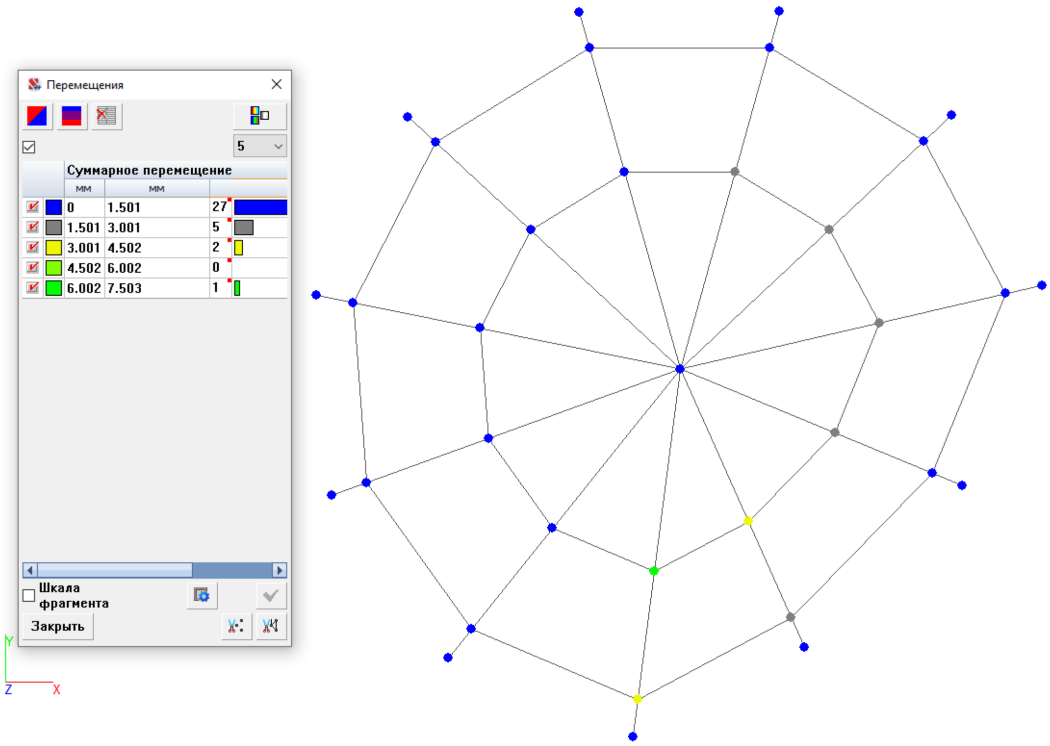

Figure 38 Total displacement